ENERGETIKA. 2025
T. 71. Nr. 1. P. 63–77
ISSN 0235-7208 eISSN 1822-8836
DOI: https://doi.org/10.6001/energetika.2025.71.1.5

Unification of approaches to the numerical solution of the boundary value problem for heat conduction using TDMA

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Modelling heat and mass transfer processes is essential in designing and optimising technological processes in power engineering, mechanical engineering, metallurgy, chemical industry, and other engineering fields. For the mathematical description of such processes, differential equations of heat conduction and diffusion are used, the solution of which requires the application of efficient numerical methods, especially in the case of complex geometries and diverse boundary conditions. This study presents a unified methodology for the numerical solution of boundary value problems of heat conduction with internal heat sources, based on locally one-dimensional implicit finite difference schemes derived using the integral-interpolation method (balance method) in Cartesian and cylindrical coordinate systems. Special attention is given to discretising boundary conditions of the first, second, and third kinds, focusing on Robin conditions, the most commonly encountered in engineering practice. A quasi-linear approximation scheme and spatial splitting schemes are recommended to increase the efficiency of numerical solutions. This approach enables the application of the unconditionally stable Tridiagonal Matrix Algorithm (TDMA). The introduction of indicator coefficients provides flexibility in implementation, allowing the balance equation to be used variably by manipulating the terms responsible for heat fluxes and the location of computational nodes. This ensures ease of implementation and improves code readability, facilitating software development for computational modelling. The results of the numerical simulation obtained using the proposed method are compared with known analytical and numerical solutions and demonstrate high accuracy. The proposed methodology opens broader opportunities for modelling thermal regimes in complex engineering systems.

Keywords: heat transfer, balance method, locally one-dimensional scheme, numerical solution, computational modelling, engineering applications

INTRODUCTION

Differential equations of heat conduction [1] and diffusion [2] play a fundamental role in modelling heat and mass transfer processes, which underlie a wide range of modern engineering technologies. These equations describe the distribution of temperature and substance concentration over

time and space, considering the physical properties of the medium under study.

The application of these equations covers a broad spectrum of tasks – from designing heat exchangers and thermal protection coatings to optimising processes such as mixing, drying, catalytic treatment, and purification of gases and liquids. In power engineering, mechanical engineering, metallurgy, chemical and food industries, as well as in construction and bioengineering, they serve as the theoretical basis for improving technological efficiency, optimisation, and ensuring environmental safety.

Like many other differential equations, these equations can be solved either analytically or numerically, depending on the complexity of the problem and the properties of the medium.

Although analytical methods provide exact solutions, their applicability is limited to simple problems with regular geometries. These methods become impractical or altogether inapplicable for cases involving complex geometries and boundary conditions [3, 4].

In real-world engineering applications, numerical methods such as the Finite Difference Method (FDM), Finite Element Method (FEM), and the Finite Volume Method (also known as the balance or integral-interpolation method) become more favourable. These methods offer flexibility and scalability when solving many practical problems [5]. Despite the inherent approximation errors and high computational costs of numerical methods, these limitations are mitigated mainly by modern hardware and advanced software implementations for computationally intensive tasks.

The first step in the discretisation process is to select an appropriate numerical method for solving a differential equation describing the modelled system. This method ultimately transforms the original differential equations into a system of algebraic equations using finite difference approximations.

These approximations are generally classified into explicit and implicit schemes. Explicit schemes are conditionally unstable, and thus suitable for short-duration processes with small time steps. Conversely, unconditionally stable implicit schemes are more optimal for simulating complex and long-term heat conduction and diffusion processes involving larger time steps, due to their lower numerical error [6, 7].

Notwithstanding the selected discretisation method – the determining factor in the accuracy and stability of the solution – the ultimate stage involves obtaining the numerical values of the target quantities (e.g., temperature or concentration fields) employing direct or iterative linear algebra methods. These methods ultimately determine the efficiency of implementing the computational model. At the same time, modern computational optimisations, including the development of parallel algorithms, can significantly reduce computational costs [8].

LITERATURE REVIEW

In physics and mathematics, the heat conduction equation is considered a particular case of the diffusion equation and represents a partial differential equation [7]. The choice of a numerical method for implementing the heat conduction equation is fundamentally essential in modelling heat and mass transfer processes, which accompany most technological operations. The optimal method selection determines the accuracy, stability, and computational efficiency of the solution, especially in the case of complex geometries and boundary conditions.

Selecting a numerical method and a discretisation approach for the original differential equation, it is typically necessary to choose between explicit and implicit finite difference schemes [6, 7]. Researchers often perform comparative analyses of solutions obtained by different methods to make informed recommendations for numerical modelling. For example, [9, 10] present comparative results for solving the heat conduction problem in a rod using explicit, implicit, and Crank-Nicolson methods (CNM). The results were compared, and the error between exact and approximate solutions was evaluated for a specific computational problem. A similar study was presented in [11], where the heat conduction equation with Neumann boundary conditions included a heat generation term. The problem was solved using the FDM in cylindrical coordinates to simulate the temperature field of a battery with time-varying heat generation. The results were compared with computations performed using built-in MATLAB functions.

Classical explicit schemes – such as the Forward Time and Centred Space and the Dufort-Frankel methods – are simple to implement and require less computational effort per time step. Nevertheless, their use is constrained by instability at larger time and spatial steps, which is governed by the Courant–Friedrichs–Lewy (CFL) condition. For instance, in the case of parabolic diffusion equations, the use of explicit schemes is generally impractical [12, 13].

Nevertheless, recent research has focused on strategies to relax – or even eliminate – these limitations, to preserve the simplicity of explicit schemes while expanding their CFL stability region [13]. Furthermore, several recent publications [2, 14–16] are devoted to developing novel unconditionally stable explicit algorithms for solving diffusion and heat conduction equations. These approaches use approximations involving constant or linearly interpolated neighbouring values instead of standard finite difference approximations. As a result of this transformation and subsequent analytical treatment of the resulting ordinary differential equations, the time step enters the formulation in exponential rather than polynomial form (with negative exponents), which fundamentally ensures the absolute stability of the algorithm.

As noted in [8, 15], achieving ideal parallelism with implicit schemes is challenging, although some progress has been made in this area, as shown in [17] for solving parabolic equations. This has renewed interest in parallelisable explicit algorithms with improved stability limits [18, 19], especially in light of the growing use of distributed and parallel computing architectures in numerical modelling [20].

Alongside the growing interest in new stable explicit schemes and the advancement of parallel algorithms for their implementation, significant attention has also been given to mathematical models involving fractional derivatives, which have gained considerable popularity in recent years [21–23]. These derivatives are increasingly applied in modelling heat transfer processes in composite and nanoscale materials and in simulating the diffusion of contaminants and moisture transport in soil or other porous media. They have proven to be an effective tool in mathematical modelling based on FDM, Laplace transform techniques, spectral decompositions, or the FEM. Particularly relevant to the current work are numerical methods for solving the fractional-order heat conduction equation [24, 25].

Compared to other numerical methods such as FDM, the FEM is used less frequently in solving heat conduction and diffusion problems. This is due to its higher mathematical complexity, implementation difficulties, and significant computational demands, especially for multidimensional problems requiring many elements. While FEM is a powerful tool for problems involving complex geometries and heterogeneous materials [26, 27], it is generally less suitable for simpler or homogeneous heat conduction and diffusion problems [28].

The Laasonen, CNM, and Alternating-Direction Implicit (ADI) methods are notable examples of implicit difference schemes. Due to their inherent stability and accuracy, these are extensively utilised for efficiently solving practical problems, including multidimensional heat conduction and diffusion [7, 29, 30]. Although all implicit methods are unconditionally stable, they may differ in the accuracy of the obtained results. The CNM, combining explicit and implicit approaches, provides second-order accuracy in time and is therefore more precise than the first-order Laasonen method. The ADI method, a locally one-dimensional scheme, is employed for two- and three-dimensional problems by decomposing the problem into a series of independent one-dimensional subproblems, significantly reducing computational complexity and enhancing understanding of multidimensional heat transfer processes [31].

Of particular note are [32, 33] that address the solution of multidimensional unsteady heat conduction equations in cylindrical and spherical coordinate systems. The five-point central difference method for solving the three-dimensional heat equation in cylindrical coordinates [32] demonstrates improved accuracy. It can be extended to develop higher-order finite difference schemes or more accurate and stable implicit schemes in cylindrical coordinates. In [33], original explicit difference schemes for heat transfer modelling in cylindrical and spherical coordinates are considered under complex boundary conditions (including convection and thermal radiation). These formulations enable heat transfer simulation in real physical

systems, whose geometries approximate those of cylinders or spheres. Understanding the characteristics of temperature distribution in different coordinate systems enables more accurate modelling of heat exchange in complex geometries and optimisation of engineering system designs concerning their operational features.

As the analytical review of the current state of the problem demonstrates, various approaches to solving heat conduction and diffusion equations have been developed over the years to account for the diversity of conditions and features inherent in specific systems. These studies span various methods and dimensionalities – from one- and two-dimensional problems to more complex multidimensional scenarios in different spatial configurations and scales [31].

Since modern modelling tasks involving heat and mass transfer processes in power engineering, mechanical engineering, metallurgy, and other sectors of the economy require the use of comprehensive approaches, where physical correctness and numerical stability are of paramount importance, implicit schemes are the most suitable to meet these requirements [29]. For the problems under consideration, implicit schemes often lead to systems of linear equations with tridiagonal matrices. In such cases, the application of the TDMA [8, 34] becomes particularly effective, offering high computational performance and efficient memory usage.

The present study aims to unify the algorithm of a locally one-dimensional TDMA-based scheme for efficiently solving boundary value problems in heat conduction (or diffusion) with variable boundary conditions, in the most commonly used coordinate systems.

RESEARCH METHODOLOGY

The nonlinear heat conduction equation of this study is the fundamental model for describing typical heat transfer processes in various media. However, it is essential to note that the methods and solutions developed for the heat conduction equation can be readily adapted to the diffusion equation, since both equations share a similar mathematical form and describe the propagation of a physical quantity within a medium (temperature and concentration of a substance, respectively) [7].

The generalised unsteady-state heat conduction differential equation, which describes the spatiotemporal propagation of heat in an isotropic and homogeneous material, can be written as follows [35]:

$$\rho(T)c(T)\frac{\partial T}{\partial \tau} = \nabla\cdot\left(\lambda(T)\nabla T\right) + q_V, \quad (1)$$

where T – temperature, K; ∇T – temperature gradient; τ – time, s; ρ – density, kg/m^3; c – specific heat capacity, J/(kg·K); λ – thermal conductivity, W/(m·K); q_V – volumetric heat generation rate, W/m^3; $\nabla\cdot$ – divergence operator.

The expanded form of this unsteady three-dimensional heat conduction equation (1) in the most commonly used coordinate systems is as follows:

in Cartesian coordinates

$$\rho(T)c(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{dx}\right) + \\ + \frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{dy}\right) + \frac{\partial}{\partial z}\left(\lambda(T)\frac{\partial T}{dz}\right) + q_V, \quad (2)$$

in cylindrical coordinates

$$\rho(T)c(T)\frac{\partial T}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(T)\frac{\partial T}{dr}\right) + \\ + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\lambda(T)\frac{\partial T}{d\phi}\right) + \\ + \frac{\partial}{\partial z}\left(\lambda(T)\frac{\partial T}{dz}\right) + q_V. \quad (3)$$

To solve applied problems of unsteady heat conduction, (2) or (3) (depending on the chosen coordinate system) must be supplemented with an initial condition, which defines the temperature distribution at the initial time in the computational domain, and with boundary conditions that describe the behaviour of temperature or heat flux at the domain boundaries. The boundary conditions are defined as follows:

First kind (Dirichlet conditions)

$$T = T_g.$$

Second kind (Neumann conditions)

$$-\lambda\frac{\partial T}{\partial n}=q.$$

Third kind (Robin conditions)

$$-\lambda\frac{\partial T}{\partial n}=\alpha\left(T-T_{\infty}\right),$$

where T_g – prescribed temperature on the boundary, K; T_{∞} – ambient temperature at the boundary, K; q – heat flux density on the boundary, W/m^2 (with $q = 0$ corresponding to an adiabatic surface); α – heat transfer coefficient, W/(m^2·K) (which is defined as the sum of several component coefficients that depend on the type of heat transfer involved – for example, combined convection and radiation [36]); n – outward normal to the boundary.

In addressing issues concerning multidimensional heat conduction, employing a locally one-dimensional scheme (i.e., a splitting scheme along spatial variables) is recommended, which combines the advantages of both explicit and implicit approaches [31]. In this case, a multidimensional heat transfer process is explicitly split into two (for planar problems) or three (for spatial problems) one-dimensional processes, which are solved sequentially using an unconditionally stable implicit scheme.

When constructing the implicit finite difference scheme for each spatial direction of the split problem, it is written in the canonical form:

$$A_iT_{i-1} - C_iT_i + B_iT_{i+1} = -F_i,\ i = 1\ldots N, \qquad (4)$$

which yields a system of N linear algebraic equations with a tridiagonal matrix, where the coefficients satisfy the conditions: $A_1 = 0$; $B_N = 0$; $A_i \neq 0$; $B_i \neq 0$.

Such boundary value problems are optimally solved numerically using the TDMA [34], for example, via the proper sweep method, which includes:

• Forward sweep (→):

$$\overset{(\rightarrow)}{\alpha}_{i+1}=\frac{B_i}{C_i-A_i\alpha_i},$$

$$\overset{(\rightarrow)}{\beta}_{i+1}=\frac{A_i\beta_i+F_i}{C_i-A_i\alpha_i},\ i=1\ldots N-1,$$

• Backward substitution (←):

$$T_N=\frac{F_N+A_N\beta_N}{C_N-A_N\alpha_N},$$

$$\overset{(\leftarrow)}{T}_i=\alpha_{i+1}T_{i+1}+\beta_{i+1},\ i=N-1\ldots 1,$$

where α and β – the sweep coefficients.

Of course, the researcher is not limited to using only the right sweep; left or counter-spread methods may also be employed [8, 34]. Moreover, counter-spread algorithms can benefit from parallel computing techniques [37].

In nonlinear heat conduction problems, the coefficients (ρ, c, λ) of (1) are temperature-dependent. Similarly, the heat generation terms (internal or surface) and the heat transfer coefficients in boundary conditions can also be functions of temperature. Nonlinear formulations most frequently arise in simulations of processes in power engineering, metallurgy, and coke-chemical industries, where temperature variations can be significant.

The author of this work advocates for the use of a quasi-linear discretisation scheme, in which temperature-dependent coefficients are evaluated based on the temperature values from the previous time layer [36].

RESULTS AND DISCUSSION

Let us consider a locally one-dimensional implicit finite difference splitting scheme for the spatial problem (2), constructed using the heat balance method on a uniform coordinate grid (Fig. 1):

$$\rho_i c_i\Delta n\frac{T_i^{\tau+\Delta\tau}-T_i^{\tau}}{\Delta\tau}=$$

$$=\frac{\lambda_{i-1/2}}{\Delta n}\left(T_{i-1}^{\tau+\Delta\tau}-T_i^{\tau+\Delta\tau}\right)-$$

$$-\frac{\lambda_{i+1/2}}{\Delta n}\left(T_i^{\tau+\Delta\tau}-T_{i+1}^{\tau+\Delta\tau}\right)+\Delta nq_V,\ i=2\ldots N-1,$$

where Δτ – time discretization step, s; Δn – grid step in the respective spatial direction x, y, or z, m.

At the boundaries of the computational domain, the following conditions may be applied:

Dirichlet conditions (temperature values prescribed T_L on the left and T_R right boundaries):

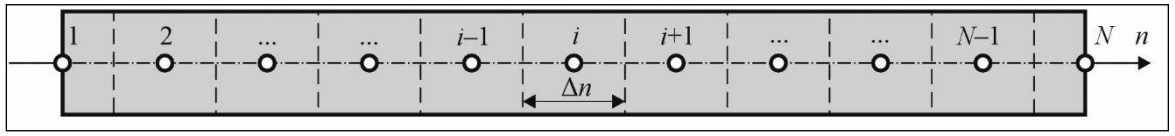

Fig. 1. Computational grid in Cartesian coordinates

$$T_1^{\tau+\Delta\tau} = T_L.$$

$$T_N^{\tau+\Delta\tau} = T_R.$$

Neumann conditions (heat flux prescribed q_L on the left and q_R the right boundaries):

$$\rho_1 c_1 \Delta n \frac{T_1^{\tau+\Delta\tau} - T_1^{\tau}}{2\Delta\tau} = q_L -$$
$$-\frac{\lambda_{1+1/2}}{\Delta n}\left(T_1^{\tau+\Delta\tau} - T_2^{\tau+\Delta\tau}\right) + \frac{\Delta n}{2} q_V,$$

$$\rho_N c_N \Delta n \frac{T_N^{\tau+\Delta\tau} - T_N^{\tau}}{2\Delta\tau} =$$
$$= \frac{\lambda_{N-1/2}}{\Delta n}\left(T_{N-1}^{\tau+\Delta\tau} - T_N^{\tau+\Delta\tau}\right) - q_R + \frac{\Delta n}{2} q_V.$$

Robin conditions (heat exchange with the environment α_L on the left and α_R right boundaries):

$$\rho_1 c_1 \Delta n \frac{T_1^{\tau+\Delta\tau} - T_1^{\tau}}{2\Delta\tau} = \alpha_L\left(T_{\infty_L} - T_1^{\tau+\Delta\tau}\right) -$$
$$-\frac{\lambda_{1+1/2}}{\Delta n}\left(T_1^{\tau+\Delta\tau} - T_2^{\tau+\Delta\tau}\right) + \frac{\Delta n}{2} q_V,$$

$$\rho_N c_N \Delta n \frac{T_N^{\tau+\Delta\tau} - T_N^{\tau}}{2\Delta\tau} =$$
$$= \frac{\lambda_{N-1/2}}{\Delta n}\left(T_{N-1}^{\tau+\Delta\tau} - T_N^{\tau+\Delta\tau}\right) -$$
$$-\alpha_R\left(T_N^{\tau+\Delta\tau} - T_{\infty_R}\right) + \frac{\Delta n}{2} q_V.$$

When splitting the unsteady heat conduction problem with respect to spatial variables, analogous finite difference equations can be written for any direction (*x*, *y*, or *z*).

Let us examine the most practically relevant case with third-kind boundary conditions (Robin conditions) imposed on all computational domain boundaries (in Cartesian coordinates, dimensionality is not essential).

Expressions for the locally one-dimensional scheme are formulated for calculating the coefficients required for solving the heat conduction boundary value problem (4) using TDMA:

At the nominal left boundary ($i = 1$):

$$A_1 = 0,\ B_1 = \frac{2\sigma_1}{\Delta n^2}\lambda_{1+1/2},$$

$$C_1 = 1 + B_1 + \frac{2\sigma_1}{\Delta n}\alpha_L,$$

$$F_1 = T_1^{\tau} + 2\sigma_1\left(\frac{\alpha_L}{\Delta n}T_{\infty_L} + \frac{q_V}{2}\right).$$

At internal grid nodes ($i = 2\ldots N-1$):

$$A_i = \frac{\sigma_i}{\Delta n^2}\lambda_{i-1/2}, B_i = \frac{\sigma_i}{\Delta n^2}\lambda_{i+1/2},$$

$$C_i = 1 + A_i + B_i, F_i = T_i^{\tau} + \sigma_i q_V.$$

At the nominal right boundary ($i = N$):

$$A_N = \frac{2\sigma_N}{\Delta n^2}\lambda_{N-1/2}, B_N = 0,$$

$$C_N = 1 + A_N + \frac{2\sigma_N}{\Delta n}\alpha_R,$$

$$F_N = T_N^{\tau} + 2\sigma_N\left(\frac{\alpha_R}{\Delta n}T_{\infty_R} + \frac{q_V}{2}\right),$$

where $\sigma_i = \frac{\Delta\tau}{\rho_i c_i}, i = 1\ldots N.$

By introducing indicator coefficients, *a* and *b*, which take values 0 or 1 depending on the position of the node on the computational grid, we can write a generalised heat balance equation for an arbitrary grid node, independent of the splitting direction and under third-kind boundary conditions on both sides:

$$
\begin{aligned}
&\left(a+b\right)\cdot\rho_i c_i \Delta n \frac{T_i^{\tau+\Delta\tau}-T_i^{\tau}}{2\Delta\tau}=\\
&=\left(1-a\right)\cdot\alpha_L\left(T_{\infty_L}-T_i^{\tau+\Delta\tau}\right)+\\
&+a\cdot\frac{\lambda_{i-1/2}}{\Delta n}\left(T_{i-1}^{\tau+\Delta\tau}-T_i^{\tau+\Delta\tau}\right)-\\
&-b\cdot\frac{\lambda_{i+1/2}}{\Delta n}\left(T_i^{\tau+\Delta\tau}-T_{i+1}^{\tau+\Delta\tau}\right)-\\
&-\left(1-b\right)\cdot\alpha_R\left(T_i^{\tau+\Delta\tau}-T_{\infty_R}\right)+\\
&+\left(a+b\right)\cdot\frac{\Delta n}{2}q_V,\ i=1\ldots N,
\end{aligned}
\tag{5}
$$

where the indicator coefficients are defined as:

$$
\begin{cases}
i=1: & a=0,\ b=1,\\
i=2\ldots N-1: & a=1,\ b=1,\\
i=N: & a=1,\ b=0.
\end{cases}
\tag{6}
$$

To solve (5) using TDMA, the general formulas are written for any node on the computational grid $i = 1\ldots N$ for computing the coefficients of (4):

$$A_i = a\cdot\sigma_i\lambda_{i-1/2}/\Delta n^2, \tag{7.1}$$

$$B_i = b\cdot\sigma_i\lambda_{i+1/2}/\Delta n^2, \tag{7.2}$$

$$
\begin{aligned}
&C_i = 1+A_i+B_i+\\
&+\frac{\sigma_i}{\Delta n}\left[\left(1-a\right)\cdot\alpha_L+\left(1-b\right)\cdot\alpha_R\right],
\end{aligned}
\tag{7.3}
$$

$$
\begin{aligned}
&F_i = T_i^{\mathrm{t}}+\sigma_i\left[\left(1-a\right)\cdot\frac{\alpha_L}{\Delta n}T_{\infty_L}+\right.\\
&\left.+\left(1-b\right)\cdot\frac{\alpha_R}{\Delta n}T_{\infty_R}+\frac{\left(a+b\right)}{2}q_V\right],
\end{aligned}
\tag{7.4}
$$

where $\sigma_i = \dfrac{2\Delta\tau}{(a+b)\cdot\rho_i c_i}$.

As previously noted, equations (7.1)–(7.4) for Cartesian coordinates are applicable for any direction in the spatial splitting of the problem. It is sufficient to replace the grid step with Δx, Δy, or Δz respectively instead of Δn.

It should be noted that the internal heat source term (if present) can be allocated to any of the spatial directions during splitting or distributed among them using specific weighting coefficients. For example, for a spatial problem, the one-dimensional equation can include a term of the form $q_V/3$.

Now, let us construct a locally one-dimensional implicit finite difference splitting scheme for the spatial problem (3), based on the heat balance method in cylindrical coordinates (r, ϕ, z) on a uniform grid. As before, we consider the practically relevant case with third-kind boundary conditions on all boundaries of the computational domain.

For the radial (Or) direction of heat transfer, the implicit finite difference scheme with third-kind boundary conditions on the inner and outer surfaces of a hollow cylinder (Fig. 2) takes the form:

$$
\begin{aligned}
&\rho_i c_i i\Delta r\frac{T_i^{\tau+\Delta\tau}-T_i^{\tau}}{\Delta\tau}=\\
&=\left(i-1/2\right)\frac{\lambda_{i-1/2}}{\Delta r}\left(T_{i-1}^{\tau+\Delta\tau}-T_i^{\tau+\Delta\tau}\right)-\\
&-\left(i+1/2\right)\frac{\lambda_{i+1/2}}{\Delta r}\left(T_i^{\tau+\Delta\tau}-T_{i+1}^{\tau+\Delta\tau}\right)+\\
&+i\Delta r q_V,\ i=K+1\ldots N-1,
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\rho_K c_K\left(K+1/4\right)\Delta r\frac{T_1^{\tau+\Delta\tau}-T_1^{\tau}}{2\Delta\tau}=\\
&=K\alpha_L\left(T_{\infty_L}-T_1^{\tau+\Delta\tau}\right)-\\
&-\left(K+1/2\right)\frac{\lambda_{1+1/2}}{\Delta r}\left(T_1^{\tau+\Delta\tau}-T_2^{\tau+\Delta\tau}\right)+\\
&+\left(K+1/4\right)\frac{\Delta r}{2}q_V,
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\rho_N c_N\left(N-1/4\right)\Delta r\frac{T_N^{\tau+\Delta\tau}-T_N^{\tau}}{2\Delta\tau}=\\
&=\left(N-1/2\right)\frac{\lambda_{N-1/2}}{\Delta r}\left(T_{N-1}^{\tau+\Delta\tau}-T_N^{\tau+\Delta\tau}\right)-\\
&-N\alpha_R\left(T_N^{\tau+\Delta\tau}-T_{\infty_R}\right)+\\
&+\left(N-1/4\right)\frac{\Delta r}{2}q_V.
\end{aligned}
\tag{10}
$$

For $K = 0$, i.e., when the computational node lies on the cylinder axis ($r = 0$), symmetry conditions are imposed in symmetric problems (Neumann conditions) – the radial temperature

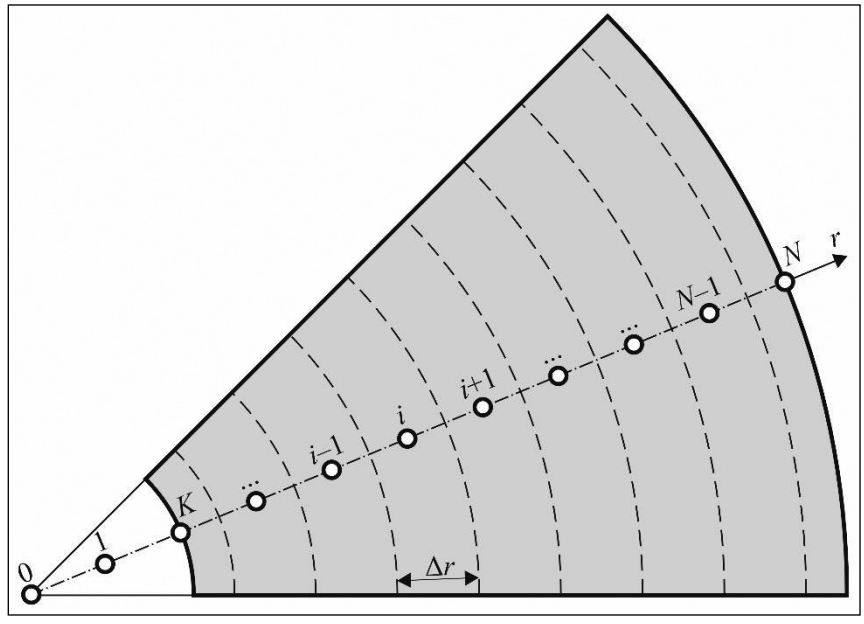

Fig. 2. Computational grid in cylindrical coordinates (radial direction)

gradient equals zero. Accordingly, in the finite difference approximation of the boundary condition, the first term on the right-hand side of (9) vanishes.

Introducing the indicator coefficients (6), we reduce (8)–(10) analogously to (5), yielding a final heat balance equation for an arbitrary node on the computational grid $i = K\ldots N$:

$$\begin{aligned}
&\left[a\cdot\left(i-1/4\right)+b\cdot\left(i+1/4\right)\right]\times\\
&\times\rho_i c_i\Delta r\frac{T_i^{\tau+\Delta\tau}-T_i^{\tau}}{2\Delta\tau}=\\
&=\left(1-a\right)\cdot i\alpha_L\left(T_{\infty_L}-T_i^{\tau+\Delta\tau}\right)+\\
&+a\cdot\left(i-1/2\right)\frac{\lambda_{i-1/2}}{\Delta r}\left(T_{i-1}^{\tau+\Delta\tau}-T_i^{\tau+\Delta\tau}\right)-\\
&-b\cdot\left(i+1/2\right)\frac{\lambda_{i+1/2}}{\Delta r}\left(T_i^{\tau+\Delta\tau}-T_{i+1}^{\tau+\Delta\tau}\right)+\\
&-\left(1-b\right)\cdot i\alpha_R\left(T_i^{\tau+\Delta\tau}-T_{\infty_R}\right)+\\
&+\left[a\cdot\left(i-1/4\right)+b\cdot\left(i+1/4\right)\right]\frac{\Delta r}{2}q_V.
\end{aligned}\tag{11}$$

To solve (11) using TDMA, we again write general equations for calculating the coefficients of (4) for any node of the computational grid $i = K\ldots N$:

$$A_i = a\cdot(i-1/2)\,\sigma_i\,\lambda_{i-1/2}/\Delta r^2,$$

$$B_i = b\cdot(i-1/2)\,\sigma_i\,\lambda_{i+1/2}/\Delta r^2,$$

$$C_i = 1 + A_i + B_i +$$
$$+i\frac{\sigma_i}{\Delta r}\left[\left(1-a\right)\cdot\alpha_L+\left(1-b\right)\cdot\alpha_R\right],$$

$$\begin{aligned}
F_i &= T_i^{\tau}+i\sigma_i\left[\left(1-a\right)\cdot\frac{\alpha_L}{\Delta r}T_{\infty_L}+\right.\\
&+\left(1-b\right)\cdot\frac{\alpha_R}{\Delta r}T_{\infty_R}+\\
&\left.+\frac{a\cdot\left(i-1/4\right)+b\cdot\left(i+1/4\right)}{2}q_V\right],
\end{aligned}$$

where $\sigma_i=\dfrac{2\Delta\tau}{\left[a\cdot\left(i-1/4\right)+b\cdot\left(i+1/4\right)\right]\rho_i c_i}$.

For the azimuthal (ϕ) direction of heat transfer, the implicit finite difference scheme with third-kind boundary conditions on both surfaces of a ring sector (Fig. 3) has the form:

$$\begin{aligned}
&\rho_i c_i R\Delta\phi\frac{T_i^{\tau+\Delta\tau}-T_i^{\tau}}{\Delta\tau}=\\
&=\frac{\lambda_{i-1/2}}{R\Delta\phi}\left(T_{i-1}^{\tau+\Delta\tau}-T_i^{\tau+\Delta\tau}\right)-
\end{aligned}\tag{12}$$

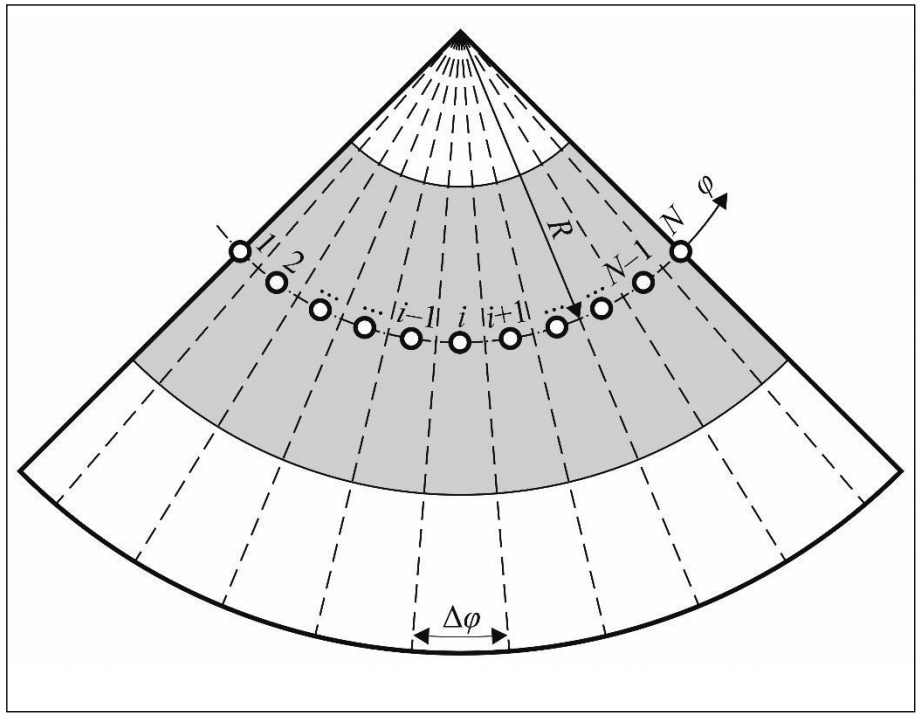

Fig. 3. Computational grid in cylindrical coordinates (azimuthal direction)

$$-\frac{\lambda_{i+1/2}}{R\Delta\phi}\left(T_i^{\tau+\Delta\tau}-T_{i+1}^{\tau+\Delta\tau}\right)+$$
$$+R\Delta\phi q_V,\ i=2\ldots N-1, \qquad (12\text{ continued})$$

$$\rho_1 c_1 R\Delta\phi\frac{T_1^{\tau+\Delta\tau}-T_1^{\tau}}{2\Delta\tau}=$$
$$=\alpha_L\left(T_{\infty_L}-T_1^{\tau+\Delta\tau}\right)- \qquad (13)$$
$$-\frac{\lambda_{1+1/2}}{R\Delta\phi}\left(T_1^{\tau+\Delta\tau}-T_2^{\tau+\Delta\tau}\right)+\frac{R\Delta\phi}{2}q_V,$$

$$\rho_N c_N R\Delta\phi\frac{T_N^{\tau+\Delta\tau}-T_N^{\tau}}{2\Delta\tau}=$$
$$=\frac{\lambda_{N-1/2}}{R\Delta\phi}\left(T_{N-1}^{\tau+\Delta\tau}-T_N^{\tau+\Delta\tau}\right)- \qquad (14)$$
$$-\alpha_R\left(T_N^{\tau+\Delta\tau}-T_{\infty_R}\right)+\frac{R\Delta\phi}{2}q_V.$$

In the case of a computational domain with a ring cross-section, i.e., closed in angle (periodic in the azimuthal direction ϕ), the coordinates of the nodes with indices $i = 1$ and $i = N$ coincide. Hence, periodic boundary conditions apply, where the temperatures at the first and last azimuthal points are equal. In the previously discussed finite difference approximation of boundary conditions, it is more convenient to apply no-flux boundary conditions, where the boundary terms in (13), (14) vanish. This can be achieved by setting the corresponding coefficient values α_L and (or) α_R to zero.

Again, introducing the indicator coefficients (6), we reduce (12)–(14) analogously to (5), yielding a final heat balance equation for an arbitrary node on the computational grid $i = 1\ldots N$:

$$(a+b)\cdot\rho_i c_i R\Delta\phi\frac{T_i^{\tau+\Delta\tau}-T_i^{\tau}}{2\Delta\tau}=$$
$$=(1-a)\cdot\alpha_L\left(T_{\infty_L}-T_i^{\tau+\Delta\tau}\right)+$$
$$+a\cdot\frac{\lambda_{i-1/2}}{R\Delta\phi}\left(T_{i-1}^{\tau+\Delta\tau}-T_i^{\tau+\Delta\tau}\right)- \qquad (15)$$
$$-b\cdot\frac{\lambda_{i+1/2}}{R\Delta\phi}\left(T_i^{\tau+\Delta\tau}-T_{i+1}^{\tau+\Delta\tau}\right)-$$
$$-(1-b)\cdot\alpha_R\left(T_i^{\tau+\Delta\tau}-T_{\infty_R}\right)+$$
$$+(a+b)\cdot\frac{R\Delta\phi}{2}q_V.$$

To solve (15) using TDMA, we write universal equations for computing the coefficients of (4) for any node on the grid $i = 1 \ldots N$:

$$A_i = a \cdot \sigma_i \lambda_{i-1/2}/(R\Delta\varphi)^2,$$

$$B_i = b \cdot \sigma_i \lambda_{i+1/2}/(R\Delta\varphi)^2,$$

$$C_i = 1 + A_i + B_i +$$

$$+ \frac{\sigma_i}{R\Delta\phi}\left[(1-a)\cdot\alpha_L + (1-b)\cdot\alpha_R\right],$$

$$F_i = T_i^{\tau} + \sigma_i\left[(1-a)\cdot\frac{\alpha_L}{R\Delta\phi}T_{\infty_L} + \right.$$

$$\left. + (1-b)\cdot\frac{\alpha_R}{R\Delta\phi}T_{\infty_R} + \frac{(a+b)}{2}q_V\right],$$

where $\sigma_i = \dfrac{2\Delta\tau}{(a+b)\cdot\rho_i c_i}$.

As seen, for the azimuthal direction of heat transfer, the implicit finite difference scheme takes the same form as (5), where $\Delta n = R\Delta\varphi$.

For the axial (Oz) direction, the implicit scheme also has the form of (5), where $\Delta n = \Delta z$.

In the problems considered above, the effective thermal conductivities of segments are computed using one of the following formulas:

$$\lambda_{i\pm1} = \frac{2\lambda\left(T_i^{\tau}\right)\lambda\left(T_{i\pm1}^{\tau}\right)}{\lambda\left(T_i^{\tau}\right) + \lambda\left(T_{i\pm1}^{\tau}\right)},$$

$$\lambda_{i\pm1} = \frac{\lambda\left(T_i^{\tau}\right) + \lambda\left(T_{i\pm1}^{\tau}\right)}{2},$$

$$\lambda_{i\pm1} = \lambda\left(\frac{T_i^{\tau} + T_{i\pm1}^{\tau}}{2}\right).$$

Thus, when modelling a heat transfer problem for a real physical system using the methodology presented here, the heat conduction equation (1) must be supplemented with boundary conditions (uniqueness conditions).

These conditions primarily include geometric specifications that define the shape and dimensions of the object under study. That is, to compute temperature fields, one must determine the dimensionality of the heat conduction equation and the coordinate system to be used. The solution is considered in Cartesian (2) and cylindrical (3) coordinate systems in the present work.

Next, physical conditions must be defined: that is, one must describe the physical properties of the material, including their temperature dependence (this information can be found in dedicated reference sources on materials and their properties) and the spatial and temporal distribution of internal heat generation, if present.

As previously noted, the quasi-linear discretisation scheme employed in this study, wherein temperature-dependent coefficients are computed using the temperature values from the previous time layer, prevents nonlinearities in the difference equations. This simplifies the numerical implementation of the algorithm for solving the resulting linear systems, enabling the efficient use of the TDMA, which overall reduces both computational complexity and runtime. Although the quasi-linear scheme does not account for instantaneous changes in the coefficients at every time step, the errors introduced due to this delayed updating are small if a sufficiently small-time step is used. Thus, the proposed scheme represents a compromise between accuracy and computational efficiency, making it particularly attractive for simulating unsteady thermal processes.

Initial conditions are also required to define the temperature distribution within the body at the initial time $\tau = 0$. In most cases, a uniform initial temperature is specified throughout all nodes of the computational domain. In the simplest case, this temperature corresponds to the ambient temperature before the onset of heat exchange.

Ultimately, it is imperative to acknowledge that the boundary conditions constitute the most critical factors in determining the adequacy of the resulting heat transfer solution, which describe the heat exchange behaviour between the surface of the body and the surrounding environment. This study focuses primarily on solving the boundary value problem for heat conduction with the most common – and at the same time most computationally demanding – third-kind (Robin) boundary conditions.

The work derives heat balance equations and expressions for calculating the coefficients in the boundary value problem of heat conduction. These formulations allow the temperature field to be calculated in bodies of arbitrary shape, approximated by surfaces of classical geometry using Cartesian and cylindrical coordinate systems.

To solve the problem numerically, a splitting method with respect to spatial variables is applied. This method enables the simultaneous solution of a multidimensional problem to be reduced to the sequential solution of several one-dimensional subproblems. At each step, the resulting one-dimensional temperature field computed using TDMA is used as the input for calculating the next subproblem.

Any such subproblem, depending on the coordinate system used, can be selected from the methodology presented. The researcher needs only to implement an iterative procedure for calculating the coefficients A_i, B_i, C_i, F_i and pass these arrays as arguments to a dedicated function for the numerical implementation of TDMA, which returns the desired temperature T_i field. The number of consecutive calls to this function equals the dimensionality of the problem.

The indicator coefficients a and b, introduced into the computational algorithm, allow for flexible use of the balance equation, selectively including or excluding specific terms related to heat flows and the position of the control node in the computational mesh. This enables the calculation of coefficients A_i, B_i, C_i and F_i within a single loop, without separating the processing of boundary node coefficients into separate blocks. As a result, the implementation becomes simpler, and the code becomes more readable, facilitating easier development, ongoing maintenance, and long-term success of the software product [38].

Figure 4 shows the results of implementing the proposed IDE Visual Studio C++ 2022 algorithm. The following input data were used to implement the heat conduction problem [28]:

One-dimensional rod of length l = 0.05 m;

Physical properties: ρ = 7200 kg/m^3, c = 544 J/(kg·K), λ = 54.42 W/(m·K);

Initial temperature: T (n, 0) = 0 K;

Dirichlet condition at the right boundary T_N = 300 K, the left boundary is thermally insulated.

A comparison of the results obtained with analytical and numerical solutions from [28] demonstrates their high accuracy (Fig. 5).

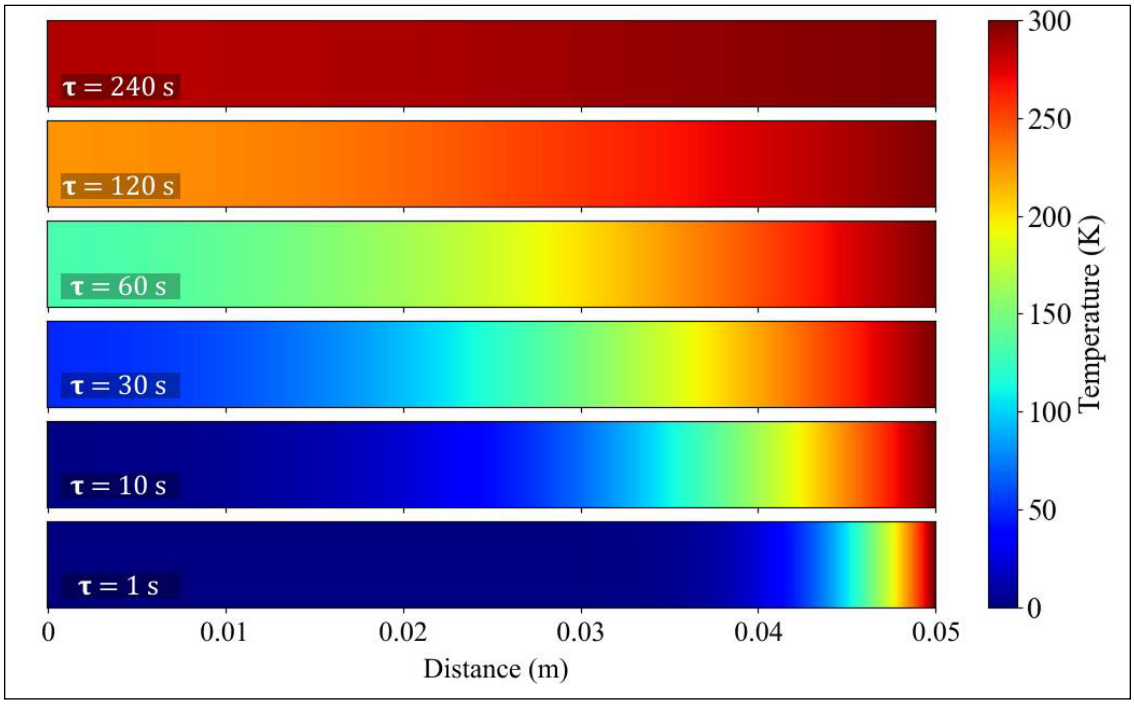

Fig. 4. Temperature profile of the rod

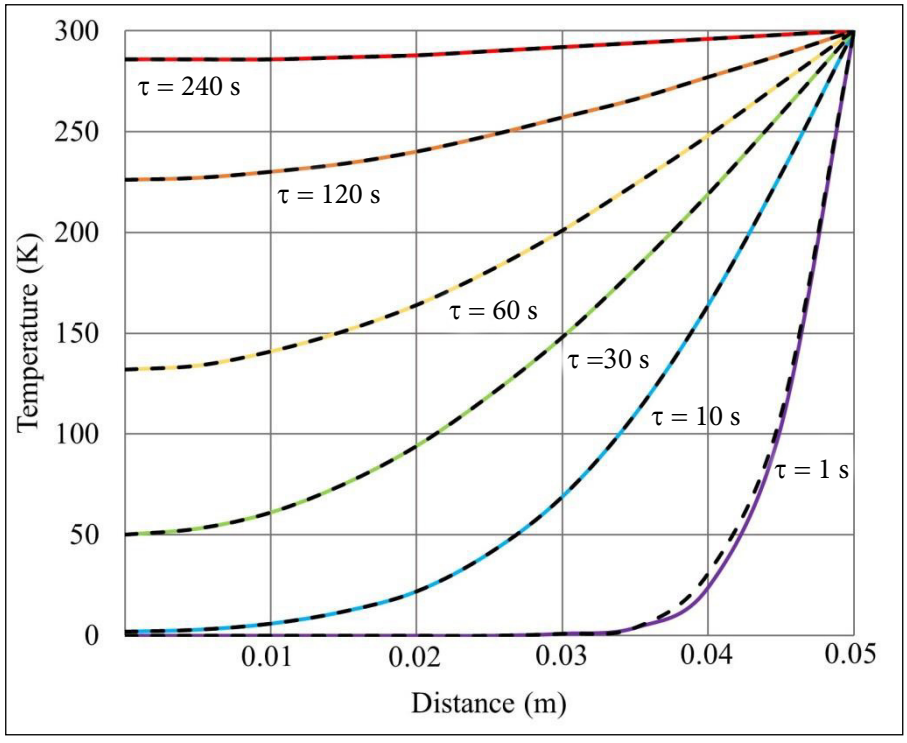

Fig. 5. Simulation data (solid lines) and data from [28] (dashed lines)

CONCLUSIONS

A numerical methodology has been proposed to solve the boundary value problem of heat conduction with an internal heat source. This approach enables the simulation of temperature fields in real physical systems, including multidimensional ones, in Cartesian and cylindrical coordinate systems. The locally one-dimensional implicit finite difference splitting schemes for the spatial problems under consideration were derived using the integral-interpolation (balance) method on a uniform coordinate grid.

Discretisation techniques for first-, second-, and third-kind boundary conditions were considered. Special attention was given to the practically important case where Robin boundary conditions are applied on all sides of the computational domain.

A quasi-linear discretisation scheme was employed, allowing for a significant simplification of the numerical implementation of the algorithm by avoiding nonlinear dependencies of material properties. This reduces computational complexity and runtime. The use of implicit approximation, together with a locally one-dimensional splitting scheme for the heat conduction equation in space, provided a foundation for employing the unconditionally stable and efficient TDMA.

The incorporation of indicator coefficients into the computational algorithm introduced additional flexibility in the selective use of the balance equations. This enables the inclusion or exclusion of specific terms affecting heat flows and, consequently, the resulting temperature distribution. Furthermore, the developed approach to software implementation of heat conduction problems unified the solution procedure based on the locally one-dimensional scheme in Cartesian and cylindrical coordinates.

The numerical simulation results obtained using the proposed methodology were compared with known analytical and numerical solutions, demonstrating high accuracy.

This study developed a unified approach to the numerical solution of boundary value problems for heat conduction (diffusion) using the TDMA. The proposed methodology opens up new possibilities for modelling thermal regimes in complex geometrical systems [39–42], commonly encountered in power engineering,

mechanical engineering, metallurgy, and other industrial sectors.

Received 29 April 2025
Accepted 26 May 2025

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ŠILUMOS LAIDUMO RIBINIO UŽDAVINIO SKAITINIO SPRENDIMO METODŲ SUVIENODINIMAS NAUDOJANT TRIĮSTRIŽAINĖS MATRICOS ALGORITMĄ

Santrauka

Šilumos ir masės perdavimo procesų modeliavimas yra svarbus projektuojant ir optimizuojant technologinius procesus energetikos, mechanikos, metalurgijos, chemijos ir kitose inžinerijos srityse. Šiems procesams matematiškai aprašyti naudojamos šilumos laidumo ir difuzijos diferencialinės lygtys, kurioms spręsti reikia taikyti veiksmingus skaitinius metodus, ypač esant sudėtingai geometrijai ir įvairioms ribinėms sąlygoms. Šiame tyrime pateikiama vieninga metodika, skirta šilumos laidumo su vidiniais šilumos šaltiniais kraštinių uždavinių skaitiniam sprendimui. Ji pagrįsta vietinėmis vienmatėmis netiesioginėmis ribinių skirtumų schemomis, sudaromomis taikant integralinės interpoliacijos (balanso) metodą Dekarto ir cilindrinėje koordinačių sistemose. Ypatingas dėmesys skiriamas pirmosios, antrosios ir trečiosios rūšies kraštinių sąlygų diskretizavimui, daugiausia dėmesio skiriant dažniausiai inžinerinėje praktikoje pasitaikančioms Robino sąlygoms. Siekiant padidinti skaitinių sprendimų efektyvumą, rekomenduojama taikyti kvazitiesinės aproksimacijos ir erdvinio skaidymo schemas. Šis metodas leidžia naudoti besąlygiškai stabilų triįstrižainės matricos algoritmą. Įvedus indikatorinius koeficientus, įgyvendinimas tampa lankstesnis, todėl balanso lygtį galima įvairiai pritaikyti, keičiant terminus, atsakingus už šilumos srautus, ir skaičiavimo mazgų vietą. Tai užtikrina paprastą įgyvendinimą ir padidina kodo skaitomumą, o tai palengvina skaičiavimo modeliavimo programinės įrangos kūrimą. Skaitinio modeliavimo rezultatai, gauti taikant pasiūlytą metodą, lyginami su žinomais analitiniais ir skaitiniais sprendimais ir rodo didelį tikslumą. Siūloma metodika atveria platesnes galimybes modeliuoti sudėtingų inžinerinių sistemų šiluminius režimus.

Raktažodžiai: šilumos perdavimas, balanso metodas, vietinė vienmatė schema, skaitinis sprendimas, kompiuterinis modeliavimas, inžineriniai taikymai