Knowability Without Rigidity

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The most straightforward interpretation of the principle of knowability is that every true proposition may be known. This, taken together with some intuitively appealing ideas, raises a problem known as the Church–Fitch paradox. There is a wide variety of alternative interpretations of the principle of knowability that have been offered to avoid the paradox. Some of them are based on rigidification of certain aspects of what is knowable. I examine three proposals representing this strategy, those by Edgington, Rückert and Jenkins. Edgington defines what is knowable as a proposition prefixed by the actuality operator. Rückert and Jenkins maintain that what makes a proposition knowable is the possibility of knowing de re (Rückert) or recognizing (Jenkins) the state of affairs that renders the proposition actually true. In both cases, the link to the actual world (or situation) rigidifies what is knowable in some aspect or other. I argue that all three theories have strongly counterintuitive consequences, and I offer an interpretation of the principle of knowability that is both free from rigidity and immune to the Church–Fitch argument.

Keywords: knowledge, knowability, the Church–Fitch paradox, proposition, state of affairs

INTRODUCTION

Are all truths knowable? Let us call the positive answer to this question the knowability principle (KP). Realists dismiss this principle, antirealists accept it, but both sides have, in order to justify their choice, to clarify its content. This is not a trivial task, because the straightforward representation of KP in epistemic logic generates the difficulty known as Church–Fitch’s paradox.¹ In the present paper, I examine some alternative logical representations of KP that were proposed to avoid the paradox, and I suggest a new representation without committing myself to the realist or the anti-realist position. Thus, my approach to the epistemological problem of knowability is based on examining some logical formalizations of KP.

¹ The paradox was first presented, in a more general form, in Fitch (1963), and many authors refer to it as ‘Fitch’s paradox’. But Fitch attributes the paradox argument to an anonymous referee of an earlier paper by him that was not published. According to Salerno (2009b), that referee was Church. That is why some authors prefer to call the paradox the ‘Church–Fitch paradox’.
The problem arises when we attempt to formally represent KP. If we take the concept of knowability on its face, it is tempting to define it as the possibility to know. In this sense, a proposition p is knowable if and only if ◊Kp is true, where ‘K’ stands for ‘someone knows at some time that’. Then KP is represented as the schema

(1) p → ◊Kp.

The Church–Fitch’s argument shows that this representation generates a serious difficulty if we accept, in addition to (1), the following intuitively appealing principles:

(2) Kp → p, and
(3) K(p&q) → Kp&Kq.

(2) says that what is known takes place. This schema, often called ‘factivity of knowledge’, immediately follows from the definition of knowledge as true justified belief. (3) says that ‘K’ distributes over conjunction, which means that if we know two things together, we also know each of them separately. Both (2) and (3) are totally harmless, but together with (1), they have an unexpected and highly counterintuitive consequence that everything is known. The argument shows that the assumption that there is an unknown truth φ leads to a contradiction and so to the conclusion that all truths are known:

(I) φ& ~Kφ assumption (with φ an unknown truth),
(II) φ& ~Kφ → ◊K(φ& ~Kφ) by substitution of (I) for ‘p’ in (1),
(III) ◊K(φ& ~Kφ) from (I) and (II) by modus ponens,
(IV) ◊(Kφ& ~Kφ) from (III) by (3),
(V) ◊(Kφ& ~Kφ) from (IV) by (2),
(VI) ~◊(Kφ& ~Kφ) a theorem of normal modal logic.

Since (I) leads to the contradiction between (V) and (VI), it must be denied:

(VII) ~φ& ~Kφ from (I), (V), and (VI).

In classical logic, (VII) is equivalent to

(VIII) φ → Kφ.

Since (VIII) has been established for arbitrary φ, it can be generalised to the schema

(IX) p → Kp,

saying that everything is known. Of course, this conclusion is not acceptable.

There is a wide variety of responses to the paradox. Some of them reinterpret KP in order to block the Fitch–Church argument; this strategy is represented by Tennant (1997; 2010), Kranvig (2006; 2010), Fara (2010), Proietti (2016), Edgington (1985; 2010), Rückert (2004), Jenkins (2007) and others. In what follows, I examine the proposal to reinterpret KP

2 Thus, two quantifiers are implicit in ‘K’. The fully explicit form of Kp is ∃x∃t (x is an agent and t is a time & Kx, tp) with ‘Kx, tp’ being a sentential operator and standing for ‘x knows at t that’.

3 Adopting quantification over propositions, we can use the formula ∀p(p → ◊Kp) instead of the schema (1). I prefer (1) because it simplifies formal details.

4 In intuitionistic logic, (VII) does not entail (VIII), which blocks the counterintuitive conclusion. According to Dummett (2001; 2009), this allows the intuitionist to accept (1) without being committed to (VIII). But it is not clear at all whether (VIII) and its generalization to the schema ~p & ~K, as well as the intuitionistically equivalent p → ~Kp, is epistemologically acceptable. For discussion on this question see Brogaard and Salerno (2006), Salerno (2009a: Part II), Rosenkranz (2004) and Murzi (2010). I leave this issue aside and stick to classical logic.

5 A purely epistemic (not involving possibility operator) version of the argument is presented in Alexander (2012). For a discussion of some related epistemological aporias see Brogaard, Salerno (2006), Fara (2012) and Borisov (2019).

6 For a review of approaches to the paradox see Brogaard, Salerno (2019).
by means of rigidifying the subject of knowledge; this proposal is represented by Edgington, Rückert and Jenkins. I show that this sort of view has strongly counterintuitive consequences and offers a version of KP that does not rigidify the subject of knowledge.

RIGIDIFICATION OF PROPOSITIONS (EDGINGTON AND RÜCKERT)

Edgington (1985; 2010) offered a solution to the paradox that has been much debated. She construes what can be knowable not as a proposition *simpliciter* but as an actually true proposition. Accordingly, KP in her version appears as follows:

\[(4) \ \text{Ap} \rightarrow \Diamond K\text{Ap} \text{ (with 'A' being the actuality operator).} \]

Notice that at each possible world, the proposition \( \text{Ap} \) has the same truth value that \( p \) has at the actual world (so any proposition of form \( \text{Ap} \) is necessarily true or necessarily false). Thus, Edgington rigidifies knowable propositions by tying them to the actual world.

This version of KP blocks the paradox reasoning, for substitution of \( \varphi \land \sim K\varphi \) for \( p \) in (4) does not lead to contradiction. On the other hand, Edgington's view was criticised for several reasons, of which I will mention two. a) Since any true proposition of the form \( \text{Ap} \) is necessarily true, what (4) renders knowable is just a subclass of necessary truths. (4) leaves aside all contingent truths, which is in disaccord with the intuitive sense of KP. b) In the consequent of (4), counterfactual knowledge is represented as knowledge that \( p \) is actually true. In terms of possible world semantics, this means that some intelligent beings in a possible world know that \( p \) holds in the actual world. But in order to have this type of knowledge, they must be able to somehow refer to the actual world even if it is distinct from the world where they exist. It is far from obvious how to bring this idea in accordance with our epistemological intuition. Both points are presented in detail in Williamson (2000: 290–301). I find both objections convincing and agree with those who claim that Edgington's proposal, if acceptable, needs substantial corrections.\(^7\)

An interesting amendment to her proposal was offered by Rückert (2004). He interprets KP using the apparatus of subjunctive logic \( S^*5 \) elaborated by Wehmeier (2005; 2012)\(^8\) and the idea of knowledge *de re*. In \( S^*5 \), predicates and quantifiers have two moods – indicative and subjunctive. Predicates and quantifiers in the subjunctive mood are marked by ‘*’; the absence of ‘*’ indicates the indicative mood. Semantically, predicates and quantifiers in the subjunctive mood are interpreted at the world of evaluation, whereas predicates and quantifiers in the indicative mood are interpreted at the actual world (which is selected in every \( S^*5 \) model), no matter what the world of evaluation is. For example, in ‘\( \Diamond \forall x P^*x \)’ the quantifier is in the indicative mood, and the predicate ‘\( P \)’ is in the subjunctive mood. Accordingly, this formula is true at \( w \) if and only if there is a possible world \( w’ \) accessible from \( w \), such that every entity in the domain of the actual world is in the extension of \( P \) for \( w’ \). Rückert's notion of knowledge *de re* can be defined as follows: to know *de re* the proposition \( p \) is to have *de dicto* knowledge implying that the state of affairs that makes \( p \) true holds. (I have slightly simplified his definition without affecting anything essential for my argument.) Now, his version of KP is this:

\[(5) \ p \rightarrow \Diamond k^*_d r e p \text{ (Rückert 2004: 373).} \]

Here ‘\( k^*_d r e \)’ stands for ‘\( \exists x \text{ (x is an agent and x knows *de re* p)} \)’. Observe that the quantifier in ‘\( k^*_d r e \)’ is in the subjunctive mood whereas the proposition ‘\( p \)’ is fully in the indicative mood.

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\(^7\) For further discussion of Edgington's proposal see Edgington (2010), Fara (2010), Schlöder (2019), Proietti (2016), Broogard and Salerno (2019).

\(^8\) In Wehmeier (2012) \( S^*5 \) is called 'subjunctive modal logic', SML.
Let us consider the example he adduces to illustrate how (5) works. He describes (Rückert 2004: 373–374) a model containing (at least) two possible worlds: the real world w and a counterfactual world w’. In w, there exist just two beings named Tom and Bob, and both are stupid in the sense that they do not know anything. Thus, the proposition that all are stupid (A) is true at w. The domain of w’ contains Tom, Bob and Jim. In w’, Tom and Bob are again stupid, whereas Jim knows of both Tom and Bob that they are stupid. Jim might express his knowledge by uttering ‘Tom and Bob are both stupid’. Of course, it is untrue at w’, and it cannot be known in w’ that all are stupid. But what Jim, in w’, knows de re of Tom and Bob is exactly what is expressed by (A) with respect to w. Thus, Jim’s de re knowledge at w’ makes (A) knowable at w⁹.

Before I present my criticism on this view, I want to notice that the example does not fully match (5), because what Jim knows de re in w’ of Bob and Tom is not that they are stupid in w – it is rather that they are stupid in w’. But, since w’ is a counterfactual world, the proposition Jim knows de re must involve the predicate of being stupid in the subjunctive mood. In the language of S⁵, the proposition under consideration is ∀xS*x, not ∀xSx (‘S’ standing for ‘stupid’). According to Rückert’s construal of KP, it should be ∀xSx → Mkₙₜ de re *∀xSx (which is an instance of (5)) that makes ∀xSx knowable at w. But according to his treatment of the example, what makes ∀xSx knowable at w is ∀xSx → Mkₙₜ de re *∀xS*x, which is not an instance of (5). So, if we take Rückert’s example as showing his intuition behind (5), and if we want KP to be in accord with that intuition (as I suppose Rückert wants it to be), we should modify the consequent of (5) to the effect that quantifiers in p are to be taken in the indicative mood, whereas predicates are to be taken in the subjunctive mood. The formal representation of this modification of (5) in general is a rather cumbersome task, so I confine myself to stating it for a special case – the case of fully indicative nonmodal truths:

(6) For any fully indicative nonmodal true sentence p, it is possible to know de re p’ where p’ is like p except that all predicates in p’ are in the subjunctive mood.

This version of the principle says that if p is a fully indicative nonmodal true proposition, there is a possible world w where someone at some time knows de re of all entities referred to or quantified over in p that they have, in w, properties ascribed to them by p. Adopting this version of KP we see that ∀xSx → Mkₙₜ de re *∀xS*x instantiates it, as it should be.

RIGIDIFICATION OF STATES OF AFFAIRS (JENKINS)

The suggested modification of Rückert’s version of KP is very close to Jenkins’s proposal (Jenkins 2007). According to Jenkins, an actually true proposition p is knowable if and only if there is a possible world w where someone recognizes that the very same state of affairs that renders p true at the actual world holds in w. Symbolizing ‘someone recognizes that the state of affairs that renders p true at the actual world holds’ as R[p], Jenkins represents KP as follows:


⁹ Kvanvig (1995; 2006; 2010) points out that the quantifiers implicit in two occurrences of ‘Kφ’ in ‘Kφ → ◊KKφ’ may have different ranges. Rückert’s example illustrates this observation: the class of agents existing in w differs from the class of agents existing in w’. Kvanvig concludes that Kφ → ◊KKφ does not instantiate KP, hence KP is safe from the Church–Fitch argument. I cannot examine his solution in detail here, so I confine myself to noticing that it does not work with respect to cases where the range of quantifiers in ‘K’ remains constant for all relevant possible worlds.

¹⁰ On p. 534 the principle is written as ‘p ⊢ R[p]’ but the absence of the diamond in the consequent is obviously a misprint as is shown by the application of the principle on p. 538.
As we see, both Rückert’s and Jenkins’s interpretations of KP rigidify the relevant state of affairs in the sense that the very same state of affairs that makes $p$ actually true is known de re, recognized to hold at the relevant possible world, and both define the knowability in terms of propositional attitudes other than knowledge. In the rest of this section I present an objection to this treatment of KP. My objection is formulated in Jenkins’s language, but it is directed against Rückert’s proposal as well.

Jenkins provides the semantic definitions of truth for sentences of the form $R[p]$, of which two clauses are important for what follows. I quote them in full:

(A) Where $p$ is $Fa$, $R[p]$ is true at $w$ iff there exists at $w$ a being who at some time recognizes, of the object $a$ which is the referent of $a$, that it has the property $F$ which is expressed by $F$ (Jenkins 2007: 534).

(G) Where $p$ is $\forall x\varphi x$, $R[p]$ is true at $w$ iff every$^{11}$ $y$ that exists at the actual world exists at $w$ and there exists at $w$ a being who at some time recognizes the obtaining of $[\varphi y]$ for every$^{12}$ such $y$ (Jenkins 2007: 535).

Following Jenkins’s treatment of restricted quantifiers (Jenkins 2007: 533), we can adapt (G) to formulae with restricted quantifiers. The clause for formulae of the form ‘$\forall C x\varphi x$’ (with ‘$C$’ a unary predicate, the formula symbolises ‘All $C$’s are $\varphi$’) results from (G) by replacing ‘every$^{*}$ $y$ that exists at the actual world’ with ‘every$^{*}$ $y$ in the extension of $C$ for the actual world’.

Applying these clauses to a sentence of the form ‘$\forall C xP x$’, we see that in order for it to be knowable, there must be an intelligent being in a possible world $w$ who recognizes of each object in the extension of $C$ for the actual world that it has the property $P$ at $w$. I want to show that this is too strong a demand.

We definitely do not know everything about atoms, and it is likely that there is a property, let us call it ‘$P$’, such that all atoms are $P$ but we do not know that. Let us refer to the proposition that all atoms are $P$ as (B). Now, is (B) knowable? According to the intuitive meaning of KP, it should be. But what does this mean on Jenkins’s proposal? It means that each actual atom exists at some possible world $w$, and some intelligent being in $w$ recognizes of each actual atom that it is $P$. If the number of atoms in the universe is infinite, and if we want (B) to be knowable, we will have to suppose that there may be an intelligent being with unlimited intellectual capacity. If the number of atoms in the universe is finite, we will have, to make (B) knowable, to suppose that there may be an intelligent being with limited but incredibly huge intellectual capacity. But both suppositions are in conflict with (nonreligious) intuitions concerning possible agents of knowledge. And what is more important, on the intuitive meaning of knowability, it should not depend on assumptions of this type. In other words, it must be sufficient for the knowability of (B) that physicists are able to discover that (B) takes place.

An even more illuminative example is the knowability of actually known truths. On the intuitive meaning of knowability, the fact that a proposition is actually known suffices for

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11 ‘Every$^{*}$’ ranges over all possible objects.

12 The formulation of (G) is slightly inaccurate in that it suggests substituting objects $y$ for the variable $x$ in $\varphi x$. It should be corrected as follows:

Where $p$ is $\forall x\varphi x$ and $v$ is a variable valuation, $R[p]$ is true at $w$ under $v$ iff every$^{*}$ $y$ that exists at the actual world exists at $w$ and there exists at $w$ a being who at some time recognizes, under $v[y/x]$, the obtaining of $[\varphi x]$ for every$^{*}$ such $y$. ($v[y/x]$ is the $x$-variant of $v$ mapping $x$ to $y$.)

It should also be noted that Jenkins assumes that individual constants designate rigidly, and each predicate letter denotes the same property (though possibly with varying extension) at all possible worlds.
it to be knowable. The proposition that all atoms consist of elementary particles is knowable just because it is known, and it is known because the relevant fact has been discovered. Why then should (B)'s knowability depend on the possibility of intelligent beings with unlimited or huge intellectual capacity? I see no reason.

Jenkins's proposal is counterintuitive not just with respect to propositions involving infinite or very large classes of objects – it is so with respect to propositions involving small classes as well. Consider the proposition that all poets are happy. (If we want, we can further restrict the quantifier by replacing ‘poets’ with ‘Russian poets’, or ‘poets born in Saint Petersburg in 1980’, etc.) On Jenkins’s view, it is sufficient for this proposition to be knowable that in some possible world w, someone recognizes of every actual poet that she is happy in w. But consider the following two cases:

1) No actual poet is a poet in w. If we are not essentialists with respect to biological species and if we allow for truly applying ‘happy’ to animals, we can even assume that some or all of actual poets are not humans in w. We can also (though we do not have to) add that not all ‘local’ poets are happy in w, so the proposition in case cannot be known in w.

2) All poets in w are actually poets, and all but one actual poet are poets in w. (In other words, the extension of ‘poet’ for w contains all actual poets but one and only them.) It is known in w that all ‘local’ poets are happy, and it is recognized of all of them that they are so.

I find Jenkins’s view counterintuitive with respect to both cases. In the first, it turns out that recognizing in w a certain fact about nonpoets suffices to make a proposition about poets knowable. In the second, recognizing in w a certain fact about all local poets (all of whom happen also to be actually poets) does not suffice to make that fact actually knowable. In both cases the knowability has nothing to do with the possibility of knowledge, which makes the theory conflict with our epistemological intuitions.

COPING WITHOUT RIGIDITY
Let us return to Edgington’s paper (1985) and consider an example she adduces to illustrate her proposal. Suppose an attempt to investigate a comet by means of a space mission is made. The comet is about to break up, so it is the last chance to investigate it. The mission can succeed or fail, which generates two possible situations – s_1 and s_2. In s_1, the mission succeeds and a piece of information about the comet – let us call it ‘p’ – has been obtained. Thus, in s_1 both p and Kp are true. In s_2, the mission fails, and, since the last chance to ascertain whether p holds has been lost, p is never known. So in s_2, we have p & ¬Kp. As a matter of fact, since the mission failed, s_2 turned out to be actual, so we have A(p & ¬Kp). Then there is possible, non-actual knowledge, in s_2, that in the possible situation s_2 (the actual situation – though it would not be described in s_1 as such) p and it is never known that p’ (Edgington 1985: 565–566). On Edgington’s view, the counterfactual knowledge in s_1 can be formally represented as KA(p & ¬Kp), which means that in s_1, someone has knowledge of s_2.

A note on the notion of situation is in order here. A situation is a fragment of a possible world. It is important for Edgington that in (4) ‘A’ refers to a situation rather than a possible world, because to be able to have knowledge of a situation, we have to somehow
identify it, and we (finite beings) can identify only fragments of worlds, not whole worlds. But the switch from possible worlds to situations does not make (4) immune to criticism. Williamson (2000) points out that an agent can identify a counterfactual situation only by providing a uniquely identifying its description. For example, in the perspective of s₁, s₂ is counterfactual, so knowledge of s₂ that someone may have in s₁ must involve the identification of s₂ by means of a description, D. To know, in s₁, that something is true at s₂, means to know that something holds in the situation uniquely described by D. But how can an agent know that? Of course, it is impossible to observe goings-on in a counterfactual situation, so the only way to get such knowledge is by deduction. In the example, someone can know in s₁ that p & ~Kp holds in s₂ only if she is aware that D entails p & ~Kp. But then the knowability of p & ~Kp reduces to the possibility of knowledge that D → p & ~Kp is logically valid. Generalising this result, we see that (4) renders knowable only logically valid propositions. Of course, KP demands more.

I find the ‘last chance to discover something’ situations like the one adduced by Edgington instructive in that they show the intuitive meaning of knowability quite clearly. Moreover, situations of this type take place very frequently, not just in science. Fara adduces an analogous example from everyday life:

For some number n it is true that there are exactly n French fries on my plate. But since no-one has counted, and since no-one will count before I finish my meal, it is also true that no-one at any time knows that there are exactly n French fries on my plate (Fara 2010: 55).¹⁴

I take this example as the starting point for presenting my interpretation of KP. Imagine an alternative to the situation described by Fara, namely a situation where an agent had counted French fries on my plate before I started my meal. In this imaginary situation the agent might truly say: ‘If I hadn’t counted the French fries on the plate, I wouldn’t have known how many of them there were.’ How to represent this piece of knowledge? Of course, the representation should reflect its counterfactual character, that is, the fact that it is knowledge of what takes place in some situations that are distinct from the situation of the agent. The most natural means of representing the counterfactual character of a proposition is the possibility operator, so this piece of knowledge can be represented as K◊p (with p standing for ‘the agent does not know that there are exactly n French fries on the plate’). Since the agent has this knowledge in a counterfactual situation, we actually have ◊K◊p, and I take this formula as the representation of knowability. Thus, I suggest representing KP as

(8) p → ◊K◊p.

Substituting φ & ~Kφ for p in (8) does not lead to contradiction, so this treatment of KP avoids the Church–Fitch paradox. And as we just saw, it neatly fits the ‘last chance to discover’ examples without invoking counterintuitive ideas like the restriction of knowability to logically valid propositions. It can also be easily verified that (8) explains all other examples of knowability mentioned above. For instance, in Rückert’s example the knowability at w of the proposition that all are stupid can be treated as Jim’s knowledge at w’ that all might be stupid. This makes (8) intuitively appealing.

Notice that (8) is completely free from rigidity. First, it does not tie knowable propositions to the actual world, as Edgington’s theory does; for every possible world w, not just the actual world, (8) renders each proposition that is true at w knowable at w. Second, it does not tie the knowability at w of a proposition p to the state of affairs that makes p true at w, as

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¹⁴ Fara’s own treatment of KP lies outside the scope of this paper.
Evgeny Borisov. KNOWABILITY WITHOUT RIGIDITY

Jenkins’s theory does; (8) says that in order for p to be knowable at w, some counterfactual agent should know that p is possible, which does not require recognizing any state of affairs. I conclude that rigidity is not needed to represent KP: knowability is nonrigid.

CONCLUSIONS
Interpretations of KP offered by Edgington, Rückert and Jenkins are all effective in blocking the Church–Fitch paradox, but each of them turns out to be in conflict with the intuitive meaning of knowability. On Edgington’s view, what makes a proposition knowable is the possibility of knowledge that the proposition holds in an actual situation, which reduces the class of knowable propositions to the class of logically valid ones. On Rückert and Jenkins’s view, a proposition is knowable due to the possibility of knowing de re the state of affairs that makes it actually true (Rückert) or of recognizing that that state of affairs holds (Jenkins). I have argued that this view conflicts with the intuition that the possibility of discovery suffices for the relevant proposition to be knowable. In addition, the Rückert–Jenkins position is counterintuitive in that it totally separates knowability from (actual or counterfactual) knowledge. All three theories share the idea that rigidity of certain aspects of propositions is essential for knowability. As an alternative, I suggested an interpretation of KP that is both free from rigidity and avoids the Church–Fitch paradox. On my interpretation, KP says that a proposition is knowable if it is possible to know that it can hold.

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References

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Santrauka


Raktažodžiai: žinojimas, pažinumas, Churcho–Fitcho paradoksas, teiginys, dalykų padėtis