

On Certain Aspects of Mathematical Objectivity¹

VLADIMIR DREKALOVIĆ

Faculty of Philosophy, University of Montenegro, Danila Bojovića bb, 81400 Nikšić, Montenegro

Email: drekalovicv@gmail.com

Physics, biology, chemistry, astronomy, psychology, sociology, history, and similar sciences that deal with empirical facts can describe the reality they investigate in varying levels of detail and provide specific statements and results about it. However, the question of describing mathematical objectivity appears to be more challenging than describing the objectivity addressed in the aforementioned sciences. This difficulty likely stems from the immaterial nature of mathematical objects, as well as their physical and temporal indeterminacy.

In this text, we will revisit one of Cantor's attempts to describe mathematical reality, select a few comments of that description, and analyse them. Additionally, we will compare Cantor's approach with some contemporary representations of mathematical Platonism that incorporate significant empiricist elements. Finally, we will propose what we believe to be a promising approach to understanding mathematical reality, one that we think offers a promising support for Platonism in mathematics.

Keywords: philosophy of mathematics, objectivity, Platonism, structuralism

INTRODUCTION

In contemporary philosophy of mathematics, we find a significant number of authors who address the issue of describing mathematical reality (Baker 2009; Colyvan 2018; Leng 2021; Pantsar 2021; Assadian, Fraser 2025; Berg 2025; Vivanco 2025). The motivation for such research is natural, as every science implies a certain objectivity (reality) that it deals with and describes. When we say *reality*, we mean the domain of entities that this science deals with. Sciences that deal with empirical facts, for example, physics, biology, chemistry, astronomy, psychology, sociology and history, can describe the reality they investigate in varying levels of detail and provide specific statements and results about it. Describing scientific reality contributes to strengthening the foundations upon which a particular science is built, helps clarify its purpose and practical aspects, and makes it more accessible and appealing to a wider audience, including those without specialised knowledge in the field.

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It is expected that the topic of mathematical reality will be addressed by mathematicians, but also by those who are interested in concrete reflections of mathematical research. The issue of describing mathematical objectivity seems to present greater challenges than describing the objectivity addressed by the aforementioned sciences. Why is this so? The most significant reason likely lies in the immaterial nature of mathematical objects, as well as their physical and temporal indeterminacy. Researchers in the aforementioned sciences can more or less precisely locate the objects of their research physically and temporally in space and time. These 'objects' can be observed directly or indirectly, or they may have been observable at some point in time.² Mathematicians, however, do not seem to have such advantages, even though mathematical reasoning is often – at both elementary and advanced levels – connected to concrete physical circumstances. For example, in the early stages of mathematical education, elementary arithmetic operations are introduced to children using physical objects as aids. We are talking, for example, about the addition of apples, marbles, etc. Similarly, complex mathematical theories, such as the Minkowski geometry, are applied in specific physical circumstances (Molinini 2016: 412–413). However, none of the aforementioned describes the general nature of mathematical objects, that is, *mathematical reality*. The previous examples are not merely *an illustration* of the connection between some mathematical objects and theories with physical reality, but also an illustration of the application of mathematical results in the circumstances of physical reality. Nevertheless, such examples do not generally characterise mathematical objectivity. They merely illustrate the relationship between *certain* mathematical and physical entities.

This paper addresses a fundamental question in the philosophy of mathematics: the nature and status of mathematical objectivity. It adopts a dual approach, combining a close analysis of selected studies on mathematical realism with a comparative evaluation of the arguments they present. In the second section, the paper examines contemporary interpretations of a Cantorian statement concerning the objectivity of numbers. It argues that Cantor's work not only inspired idealist conceptions of mathematical theory but also laid the groundwork for perspectives that diverge sharply from idealism. Within this context, the discussion further explores various strands of mathematical Platonism, with particular attention to the nuanced position of *subtle* Platonism. The third section turns to structuralist accounts of mathematical objectivity, offering both an analysis and a critical assessment of *strong* structuralism. The paper concludes by advocating *moderate* structuralism as a theoretically coherent and philosophically compelling framework for understanding the objectivity of mathematics.

CANTOR'S DESCRIPTION OF MATHEMATICAL OBJECTIVITY, SOME KEY POINTS AND PLATONISM

One of Cantor's views on mathematical reality is inevitably, often quoted and analysed:

First, we may regard the whole numbers as real in so far as, on the basis of definitions, they occupy an entirely determinate place in our understanding, are well distinguished from all other parts of our thought and stand to them in determinate relationships, and thus modify the substance of our minds in a determinate way (Cantor 1883: 562).

² Certain phenomena, such as those accessible through experimentation, can be temporally shaped by the researcher. In contrast, other events – whether particular occurrences in the cosmos or historical developments within social communities – possess an objective temporal existence independent of any researcher's interventions. They occurred at determinate points in the past or will inevitably occur in the future.

In the literature, various interpretations of the quote above can be found. For example, Tait suggests that the study of mathematics involves a psychological perspective on concrete mathematical objects. The key idea conveyed is that numbers have an underlying reality, not in a physical sense, but as a coherent system that describes the relationships between them. In other words, the reality of numbers lies in the consistent and structured way they relate to one another, rather than in any tangible existence. The objectivity of mathematical objects should not be understood as the description of the mode of existence of mathematical entities or as the existence of Plato's world in which such objects would already reside, but rather as the *objectivity of mathematical discourse* (Tait 2001: 22). Similarly, Pantsar maintains that mathematical reality is not represented by the existence of some platonic world of mathematical objects, but by the characteristics of mathematical *thinking* that should be the standard for objectivity (Pantsar 2021: 321).

It seems that authors, relying on Cantor, explicitly remove the idea of mathematical objectivity from any ontological context in which one could speak of the usual understanding of the words *reality* and *objective* existence of a particular set of entities – entities whose existence would be independent of individual thinkers or the thinking, results, and cognition of a scientific community.³ In this way, as a consequence of the preceding analysis, we can say that both authors reject the metaphysical idea of the existence of a special Platonic world of objects because such a world, in its hierarchy, among other things, implies the existence of mathematical objects. Rejecting the possibility of such a mathematical realm as a substructure of Plato's world also entails rejecting Plato's world.

Based on the quoted text, it appears undoubtedly clear that Cantor, when characterising mathematical objectivity, excludes any metaphysical constructions, such as those derived from Plato. In Cantor's view, *mathematical objectivity* is a collective stance of the scientific community – a set of truths expressed by well-defined concepts and supported by provable or postulated statements. However, unlike Tait, we believe that it is *thought*, rather than *discourse* about objects, that constitutes the space in which Cantor places mathematical reality. Tait's interpretation of the concept of objectivity through the lens of discourse is likely driven by the fact that, in Cantor's sense, the characterisation of objectivity implies some form of verification of the statements that describe such objectivity. Such verification is carried out by members of the mathematical community and can be achieved solely through a form of communication and discourse among them. In this regard, some type of discourse would be indispensable for the recognition of this kind of objectivity. Nevertheless, such discourse would function merely as a technical instrument for the transmission and exchange of conceptual content among the members of the community. On the basis of such reflections, the generally accepted positions of the community would be formed. The totality of these positions would constitute objectivity. It also seems to us that Pantsar is mistaken when, in analysing Cantor's views, he claims that mathematical reality consists of the *characteristics* of mathematical thinking. We believe that it would be more appropriate to speak of the *content* of mathematical thinking. Rather, it is precisely the content of such thinking that embodies mathematical objectivity, and this content is shaped by a specific deductive methodology characteristic of mathematical reasoning.

³ When discussing the objective existence of certain entities, most contemporary authors addressing epistemological and ontological issues primarily refer to a material-physical type of existence that can be verified through specific causal or similar procedures. Unfortunately, this perspective is very common even in the philosophy of mathematics, where causality has gained a significant importance, even in relation to mathematical knowledge (Benacerraf 1973; Maddy 1990).

It seems to us that mathematical reality is accurately described by mathematical practice. Namely, not only numbers, but also any other defined mathematical objects and their properties, which are expressed by various provable statements, are part of the mathematical world. The rigor of mathematical methodology allows us to claim that we can distinguish each of these objects precisely from one another, as well as from non-mathematical entities. This exactness also allows us to examine the relationships between such different objects through various statements. In this way, through continuous development, mathematics deductively constructs, complements and enriches its own objectivity. However, one question remains open: Is the evolving thought of the mathematical community – refined through the construction of deductive theories – the result of discovering something that already exists as a potential to be uncovered (an objective reality)? Or is the construction of deductive structures independent of any *real* possibilities, relying instead on the arbitrary, subjective and constructive powers of the mathematical community? In other words, is the development of mathematics a process of discovering pre-existing entities and structures, or is it a non-material, unconstrained creation with no connection to an external, non-subjective reality? Is it a matter of constructing objectivity, the discovery of which is a privilege of gifted researchers, where the fate of such objectivity is determined solely by the research, capabilities and motives of such individuals? It seems that Hilbert's answer to the last question would be affirmative. Namely, according to Hilbert '... No one shall drive us out of the paradise which Cantor has created for us...' (1926: 191). Thus, Hilbert considers Cantor's mathematical creation to be a paradise – a subjective and idealistic construct, built independently of any external, objective reality. However, not all authors subscribe to such an idealised conception of mathematical reality. Wittgenstein, for example, states that 'I wouldn't dream of trying to drive anyone out of this paradise. I would try to do something quite different: I would try to show you that it is not a paradise ...' (Wittgenstein 1939: 103).

Wittgenstein regards only applied mathematics as meaningful, while unapplied mathematics is merely a game with symbols: '... a piece of mathematical architecture which hangs in the air, and looks as if it were, let us say, an architrave, but not supported by anything and supporting nothing ...' (1933/44). Anything beyond such mathematical content is frequently marginalised, as supported by some authors such as Maddy who claims:

The prose, the fog, the mistaken philosophical theorizing that Wittgenstein undertakes to excise from mathematical discourse is platonism: the view that mathematics is the study of a non-spatio-temporal realm of abstract entities (Maddy 1997: 166).

Can we accept the claim that mathematical Platonism is merely empty theorising? To explore this question, let us recall one characterisation of *mathematical Platonism*. According to this view, mathematical Platonism entails three conditions regarding mathematical objects:

- a) *the existence* of mathematical objects;
- b) *the abstract nature* of mathematical objects;
- c) *the independence* of mathematical objects from our language, thought and practices

(Linnebo 2018) (1).

One of the crucial points in the recent history of the philosophy of mathematics is the search for the foundation – the objectivity – upon which the conceptual and formally deductive structure of mathematics is built. In this context, numerous attempts have been made to *rescue* the position of Platonism in the recent past. In fact, there has been a search for objectivity that would serve as the content to which mathematical theories would refer. Some of these attempts insist on a causal connection between the subject of cognition and mathe-

mathematical objects (Benacerraf 1973). The second type of attempt can be considered more subtle, as it does not insist on an explicit causal connection in the acquisition of knowledge about mathematical objects. Nevertheless, it sought to objectify mathematical entities in a sophisticated way – by appealing to material circumstances, to the extent that Platonism was equated with realism (Maddy 1990). The third type of attempt offered an even higher level of empirical support for Platonism in mathematics. Specifically, it aimed to construct an argument from which the central claim of mathematical Platonism – that mathematical objects exist – would follow. In essence, the argument is this: if mathematical objects are indispensable explanatory tools in science, and we take the existence of scientific entities to be unquestionable, then the existence of those tools – i.e. mathematical objects – must also be considered unquestionable (Baker 2009: 613).

Roughly a decade ago, another attempt was made to articulate a Platonist account of mathematical reality. In his argumentation, Rayo endeavoured, among other things, to disentangle this account from the causal, material and empiricist elements characteristic of earlier versions, thereby more readily satisfying the requirement of abstractness stipulated in definition (1). The author proposes designating this approach as *subtle Platonism* (Rayo 2015: 83). The term reflects the manner in which mathematical objectivity is conceived. In this view, the world of mathematical entities is neither embedded in the physical world nor invoked as a tool for explaining empirical reality. Instead, subtle Platonism offers an alternative route, encapsulated in the claim that mathematical objects – such as numbers – exist, but only in a *trivial* sense. This triviality, Rayo argues, follows from logical necessity and can be formally demonstrated as a theorem. In other words, it is logically necessary that numbers exist, because a world without them would be inconsistent (Rayo 2015: 81). In other words, the triviality of the existence of numbers does not stem from some self-evident truth accessible to common sense, nor does it arise from an obvious empirical fact perceptible to all individuals, regardless of their mathematical aptitude. Rather, the triviality of the existence of natural numbers follows from a theorem that is a direct consequence of the generalisation of statements about the physical world. Specifically, Rayo argues that from

'For the number of dinosaurs to be zero *just is* for there to be no dinosaurs.' (2)
by generalising, we get

'For the number of *Fs* to be *n* *just is* for it to be the case that $\exists!_n x(Fx)$ ' (Rayo 2015: 81) (3).

From this, it trivially follows – by *reductio ad absurdum* – that a world without numbers would be inconsistent. Namely: assume, for *reductio*, that there are no numbers. By (3), 'for the number of numbers to be zero *just is* for there to be no numbers. So, the number of numbers is zero. Therefore, zero exists. Hence, a number exists. Contradiction' (Rayo 2015: 81).

In this way, subtle Platonism – at least with respect to one type of mathematical object – satisfies all the conditions outlined in definition (1) at the beginning of this section. The existence of numbers is guaranteed independently of human thought, and unlike the earlier versions of Platonism, the condition of abstractness is also fulfilled. More precisely, mathematical objects are not required to exhibit any form of causality, physical characterisation, or explanatory role within the empirical sciences. The mention of biological beings and the concrete world in statement (2) merely serves to illustrate the more general principle articulated in statement (3), which provides one way of capturing the general nature of certain mathematical objects. The originator of subtle Platonism argues that 'there is *no difference* between there being no dinosaurs and the number of dinosaurs being zero', just as there is no difference between drinking a glass of water and drinking a glass of H₂O (Rayo 2015: 81).

According to (3), we can similarly say that there is no difference between there being n things that possess property F and the number of F s being n . Thus, according to subtle Platonism, the use of entities from the physical world serves only as an illustration of the general ontological status of mathematical objects – in this case, numbers. The claim that a connection between mathematical entities and concrete reality is necessary for defending Platonism does not apply here. In this framework, *being a dinosaur* is treated as just one property among many that can be defined, and neither it nor any other concrete property holds fundamental significance – only illustrative value. Physical entities are mentioned solely to demonstrate, in specific cases, the systemic or structural position of certain mathematical objects in relation to empirical reality.

AN ATTEMPT AT A SOLUTION

Subtle Platonism could serve as a starting point for a new way of understanding mathematical objectivity. Specifically understanding the role of numbers, as presented in this form of Platonism, might offer a reliable model for understanding the status of mathematical entities beyond just numbers. Numbers are analysed in relation to the world in which they can be observed: they describe the quantity of certain objects in that world. In this way, a degree of similarity or structural correspondence can be found between mathematical objects and objects in the physical world. However, even when we associate a specific number with a particular set of physical objects, we cannot claim that this association captures the objectivity of the number itself, its presentation in reality, or anything of that kind. Such an association is merely an illustration of the *structural* relationship between a specific number and all sets of objects to which that number can be assigned. This is a structural link, because the nature of the mathematical object – in this case, a number – captures a structural aspect of the sets in question (whose elements are entities in the physical world): namely, their cardinality. Thus, we could say that the objectivity of such a number is grounded not only in its abstractness but also in its generality, which extends across an infinite range of individual entities.⁴ Each of these entities can be grouped into sets to which the same number applies. The number refers, among other things, to all such entities. The totality of these entities may be viewed as part of the objectivity of that number. As we stated in the introductory part of the article, mathematical entities and theories sometimes have their illustrations and applications in specific cases of the empirical world. However, such individual cases do not describe mathematical objectivity. It is described by the *structural identities* that connect all such individual empirical cases. There can be infinitely many such cases.⁵

Let us begin with a general observation. In the physical world, we encounter various phenomena that can be described both as sets of specific objects and as sets of relationships between those objects. Each such phenomenon can thus be regarded as a composite entity with a distinct structure – a structure that can often be identified at a general level even among entirely different physical entities, including those that differ significantly from the ones originally used to formulate the structural concept. These general structures appear in mathematics as *meta-objects* or as components of mathematical theories. In other words, in such cases,

⁴ For the number of dinosaurs to be zero *just is* for there to be no dinosaurs. But the same applies to any other extinct or imaginary creature.

⁵ I would like to thank the anonymous reviewer for drawing my attention to the need to clarify the role of empirical cases in the description of mathematical objectivity.

a mathematical structure can be recognised as the underlying structure shared by a wide range of configurations within the physical world.⁶ In extreme forms, this view is endorsed by some contemporary authors, such as Mary Leng (Leng 2021). In accordance with her views, let us refer to her conception as *strong structuralism*, mathematical and physical structures correspond to each other *in their entirety*. More precisely, *all* physical structures and their instances in the physical world are instances of structures found in mathematics. Furthermore, such research aims to show that mathematical explanations of empirical phenomena are possible thanks to the structural properties of mathematical theories whose instantiations are concrete phenomena of the physical world. In other words, structural explanations explain a phenomenon by showing it to have been an inevitable consequence of the structural features instantiated in the physical system under consideration (Leng 2021: 10421). This would further imply that for any structure of the physical world F_1 there exists a corresponding mathematical structure M_1 such that F_1 is an instantiation of M_1 and that there exists some kind of bijective mapping between F_1 and M_1 . In the physical world, we can find an infinite number of structures F_1, F_2, F_3, \dots where each of them is an instantiation of M_1 . This could imply the idea that all structures of the physical world are in a certain way prefigured in mathematical structures. That is, every structure of the physical world possesses its own bijective counterpart within the domain of mathematical conceptuality.

We could not entirely agree with the ideas of strong structuralism, for several reasons. First, it seems implausible to provide either a formal or an informal proof for the claim that all structures found in the physical world correspond to general mathematical structures. The reason is trivial: it is impossible to systematically catalogue all mathematical and physical structures – especially mathematical ones – since they are continually being developed and expanded. We cannot even begin to predict the full scope of mathematical theories in the future. Second, the idea of structural instantiation implicitly introduces a hierarchical relationship between the two types of structures. That is, physical structures would, in a sense, be ‘beneath’ mathematical ones, since their identification in nature would be understood as mere instantiations of the latter. If such a hierarchy were to exist, it would naturally raise questions about the nature and origin of this relationship, and about the source of the authority or ‘power’ that mathematical structures would possess in such a framework. It seems that any attempt to answer these questions would inevitably lead to metaphysical, rather than analytical, arguments.

CONCLUSIONS

A strong structuralist approach to the question of mathematical objectivity entails a belief in ‘preordained’ mathematics, according to which mathematical structures, through the configuration of their theories, constitute generalisations of *all* structures that can be found in the physical world. However, the fact that we can identify structural properties from the graph theory in the layout of the bridges of Königsberg, or that we recognise statements from the number theory in the life cycle of periodical cicadas in North America, does not serve as evidence that a mathematical structure or regularity can be found in *all*

⁶ There are well-known examples in which a concrete physical structure is recognised within a more general model of a mathematical structure. For instance, the essence of the Seven Bridges of Königsberg problem or the life cycle of North American cicadas can be addressed using theorems from the Graph theory or the Number theory (Colyvan 2018).

empirical phenomena. For example, it is difficult to believe that we have any formal guarantee that a mathematical model can be applied to everyday events such as the boiling of water in a kitchen or the way the lizard *Phyllurus fimbriatus* climbs the rocks of Scawfell Island in Australia. However, what we can reasonably expect from a moderate structuralist approach to mathematical objectivity is the justified recognition of mathematical structures either as generalisations of certain empirical structures or as less general mathematical forms that have already been shown to generalise some aspect of the physical world. In this way, a meaningful and realistic path can be charted – one that offers justified hope for assembling arguments and evidence that, taken together, form a coherent picture of mathematical objectivity. At the same time, this approach helps to avoid the pitfalls that would lead either to a causal and physical-materialistic description of mathematical reality, on the one hand, or to mystical metaphysics, on the other.

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VLADIMIR DREKALOVIĆ

Apie kai kuriuos matematinio objektyvumo aspektus

Santrauka

Fizika, biologija, chemija, astronomija, psichologija, sociologija, istorija ir kiti empirinius faktus nagrinėjantys mokslai gali įvairiu detalumo lygiu aprašyti tiriamą realybę ir pateikti konkretius teiginius bei rezultatus. Tačiau matematinio objektyvumo aprašymas atrodo sudėtingesnis nei minėtų mokslų nagrinėjamo objektyvumo aprašymas. Šis sudėtingumas greičiausiai kyla dėl matematinių objektų nematerialios prigimties, taip pat dėl jų fizinio ir laikinio neapibrėžtumo.

Šiame tekste apžvelgiamas vienas iš Georgo Cantoro bandymų aprašyti matematinę realybę, parenkami keli to aprašymo komentarai ir jie analizuojami. Taip pat G. Cantoro požiūris lyginamas su kai kuriais šiuolaikiniais matematinio platonizmo vaizdiniais, kuriose esama reikšmingų empirinių elementų. Galiausiai pasiūlomas perspektyvus požiūris į matematinės realybės supratimą, suteikiantis daug žadančią atramą platonizmui matematikoje.

Reikšminiai žodžiai: matematikos filosofija, objektyvumas, platonizmas, struktūralizmas