

DIE HARD HOLOGRAPHIC PHENOMENOLOGY OF CUPRATES

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We discuss the attempts of fitting a number of the approximate power-law dependences observed in the cuprates into one consistent holographic or holographically inspired hydrodynamic framework. Contrary to the expectations, the goal of reproducing as many as possible of the established behaviours of the thermodynamic and transport coefficients appears to be achievable within the picture of a non-degenerate fermion fluid with quadratic dispersion. While not immediately elucidating the essential physics of the cuprates, this observation suggests a possible reason for which the previous attempts towards that goal have so far remained inconclusive.

Keywords: high- T_c cuprates, applied holography, transport, hydrodynamics

1. Transport in cuprates

The normal state of the cuprate superconductors has long remained a challenge defying many attempts of its theoretical understanding. In the continuing absence of a fully satisfactory microscopic description, a modest goal has been that of constructing a more or less successful phenomenological description capable of accounting for most of the observed transport properties.

Initially, the phenomenologies of the cuprates focused on the much publicized dichotomy between the robust power-law behaviours of the longitudinal conductivity observed in the optimally doped YBCO (and, to a lesser extent, LSCO) compounds

$$\sigma \sim T^\alpha \quad (1)$$

with $\alpha_{\text{exp}} = -1$ and the Hall angle

$$\tan \theta_H \sim T^\beta \quad (2)$$

manifesting the exponent $\beta_{\text{exp}} = -2$.

In the early theoretical proposals, Eqs. (1, 2) were argued to imply the existence of two distinct scattering times, $\tau \sim T^{-1}$ and $\tau_H \sim T^{-2}$, which were supposed to characterize the relaxation of either longitudinal vs transverse [1], charge-symmetric vs antisymmetric [2] currents, or a two-fluid nature of charge and heat transport [3]. Yet another insightful proposal of the ‘marginal Fermi liquid’ phenomenology was put forward early on [4].

Additional evidence of anomalous transport in the cuprates was provided by the magnetoresistivity

$$\frac{\Delta\rho}{\rho} \sim T^\gamma \quad (3)$$

that violates the conventional Kohler’s law $\Delta\rho/\rho \sim B^2/\rho^2$, instead featuring the exponent $\gamma_{\text{exp}} = -4$ [5] (in strong fields $B \gg T$ the quadratic field dependence changes to a linear one).

The anomalous transport properties (1–3) appear to coexist with the fairly conventional thermodynamic ones, including the Fermi-liquid-like specific heat and entropy

$$c \sim s \sim T^\nu \quad (4)$$

with $\nu_{\text{exp}} = 1$ [6]. In some compounds, upon approaching the pseudogap phase, $c(T)$ can also show a logarithmic enhancement, possibly signifying a quantum phase transition [7].

In the presence of thermal gradients, the combined thermo-electric response is described by the coefficients relating the charge \mathbf{J} and heat \mathbf{Q} currents to the gradients of electric potential and temperature,

$$\begin{aligned} \mathbf{J} &= \hat{\sigma} \mathbf{E} - \hat{\alpha} \nabla T, \\ \mathbf{Q} &= T \hat{\alpha} \mathbf{E} - \hat{\kappa} \nabla T, \end{aligned} \quad (5)$$

where the 2×2 matrices such as, e.g. $\hat{\sigma}_{ij} = \sigma \delta_{ij} + \sigma_H \epsilon_{ij}$, are composed of the longitudinal and transverse (Hall) components.

Early on the studies of heat transport focused on the Hall component as its longitudinal counterpart is believed to be dominated by the phonon contribution in most of the phase diagram [8, 9]. However, in the presence of chiral spin structures a sizable κ_H signal might also stem from phonons or magnons, or even both [10].

The list of the actually measured observables includes the thermopower (Seebeck) coefficient, thermal conductivities at zero current, the Hall Lorenz number, and Nernst coefficient

$$\begin{aligned} S &= \alpha/\sigma, \quad \kappa = \bar{\kappa} - \alpha^2/\sigma, \\ L_H &= \kappa_H/T\sigma_H, \quad e_N = \frac{\alpha_H \sigma - \alpha \sigma_H}{\sigma^2 + \sigma_H^2}. \end{aligned} \quad (6)$$

For some of these quantities the available data still remain scarce and their independent verification is badly needed. Nonetheless, the above coefficients (in the case of thermopower, its deviation from a possible constant term) might also exhibit the power-law dependences

$$\begin{aligned} \kappa_H &\sim T^\delta, \quad L_H \sim T^\lambda, \\ e_N &\sim T^\mu, \quad S \sim T^\rho, \end{aligned} \quad (7)$$

where $\rho_{\text{exp}} \approx 1$, $\delta_{\text{exp}}, \mu_{\text{exp}} < 0$, and $\lambda_{\text{exp}} \geq 0$.

More specifically, in Refs. [8, 9] the data on L_H in the untwinned samples of optimally doped

YBaCuO were fitted into a linear dependence ($\lambda = 1$) while the Nernst signal e_N was found to increase dramatically with decreasing temperature. This effect was attributed to the superconducting fluctuations and/or disordered vortex pairs whose (positive) contribution dominates over that of the quasiparticles (whose sign, in turn, depends on the dominant type of carriers) upon approaching T_c . Besides, e_N turned out to be strongly affected by a proximity to the pseudogap regime and can even become anisotropic [11–14].

However, the subsequent Refs. [15–17] reported somewhat different results for σ_H and κ_H , and the concomitant slower temperature dependence of L_H in the LaSrCuO, EuBaCuO and YBaCuO compounds. Specifically, in the twinned YBaCuO samples the measured exponents were

$$\beta_{\text{exp}}^{[15-17]} = -1.7, \quad \delta_{\text{exp}}^{[15-17]} = -1.2, \quad \lambda_{\text{exp}}^{[15-17]} = 0.5. \quad (8)$$

Unlike Refs. [8, 9], the measurements in Refs. [15–17] of both σ_H and κ_H were carried out on the same, rather than different, samples.

Adding to the puzzle of the cuprates' transport properties, there have been persistent reports of the Fermi liquid-like rate of inelastic quasiparticle scattering [18–22, 25]

$$\Gamma_{\text{qp}} \sim T^2, \quad (9)$$

in contrast to the almost uniformly accepted [23] and seemingly ubiquitous (see, however, Ref. [24]) 'Planckian' dissipation rate that is believed to control local equilibration/thermalization

$$\Gamma_{\text{eq}} \sim T. \quad (10)$$

Generally, the latter would be expected in a quantum-critical phase associated with a quantum phase transition and in the absence of an intrinsic energy scale, other than temperature.

In the context of the cuprates, a number of the potentially viable quantum critical transitions have been discussed, their list including superconducting, spin, charge, nematic, as well as other, even more exotic, instabilities. However, some data [25] indicate that the quantum critical scenario may not necessarily be at work.

Nevertheless, the belief in the universality of Eq. (10) and its interpretation as a key evidence in

support of the strong (as opposed to just moderate) correlations in the cuprates has brought to life a number of proposals based on the various ‘ad hoc’ generalizations of the original ground-breaking conjecture of holographic correspondence.

2. Applied holography

In its own words, the ‘bottom-up’ applied holography (a.k.a. AdS/CMT or anti-deSitter/condensed matter theory correspondence) purports to offer a unique, intrinsically strong-coupling, approach to a variety of the traditionally hard condensed matter problems [26–32]. On the technical side, this intriguing (albeit still lacking any solid proof) scheme borrows its computational apparatus (in essence, ‘ad verbatim’) from the original machinery of the conjectured holographic AdS/CFT (anti-deSitter/conformal field theory) correspondence which was developed and professed in the ‘bona fide’ string/field theory.

From the conceptual standpoint, searching for a common cause of the observed properties would indeed make perfect sense if the sought-after universality were indeed present. However, under a closer inspection even some close members of the same family of materials often demonstrate different behaviours and exhibit different power laws. Obviously, any significant diversity between the related compounds would be rather difficult to accommodate under the holographic paradigm, since virtually every compound would then require individual treatment and a material-specific dual bulk geometry.

Such potential difficulties notwithstanding, the decade-long vigorous work on the AdS/CMT exploited a variety of the classical geometries (Reissner–Nordström, Lifshitz, hyperscaling-violating, Bianchi, Q-lattices, etc.) [26–32]. Such exploratory studies resulted in a number of rather exotic proposals for obtaining some of the exponents (1–4, 7), although in order to reproduce even the basic Eqs. (1, 2) such analyses would often have to go to a great length.

To that end, one popular scheme [33] invokes the extreme ‘ultra-local’ AdS₂ limit, where both the dynamical critical index z and the ‘hyperscaling-violation’ exponent θ take infinite values, thereby conspiring to make the conductivity inversely proportional to the entropy density

$$\sigma \sim T^{(\theta-2-z)/z} \sim s^{-1} T^{-2/z} \quad (11)$$

in order to conform to Eq. (1) for $z \rightarrow \infty$.

Another, more comprehensive, attempt was made in Ref. [34] where the values $z = 4/3$ and $\theta = 0$ were argued to reproduce the observed behaviour of the transport coefficients (1–3) while $\lambda_{\text{holo}}^{[34]} = 1$ was chosen as one of the constitutive relations (despite the fact that, unlike Refs. [8, 9], the works of Refs. [11–17] reported a slower-than-linear in – or even decreasing with – T electronic Lorenz ratio), yet still other exponents

$$v_{\text{holo}}^{[34]} = 1.5, \rho_{\text{holo}}^{[34]} = 0.5, \mu_{\text{holo}}^{[34]} = -1.5 \quad (12)$$

were markedly off their targeted values (4), and also

$$\rho_{\text{exp}} = 0 \text{ or } 1, \quad (13)$$

depending on whether the goal was to fit the constant (which may or may not have been of electronic origin) or the linear term in the experimental plot for LaSrCuO [35]

$$S = a - bT. \quad (14)$$

In that regard, the analysis of Ref. [34] ignored the constant and went straight for the T -dependent term.

For comparison, the alternate scaling scheme of Ref. [36] characterized by the least exotic values $z = \theta = 1$ readily produces the exponents Eqs. (1–4), alongside

$$v_{\text{holo}}^{[36]} = 1, \rho_{\text{holo}}^{[36]} = 1, \quad (15)$$

and, to a lesser extent, $\lambda_{\text{holo}}^{[36]} = 0$. All of the above appear to generally agree with experiment (there was not enough data available for ascertaining the predicted values of $\mu_{\text{holo}}^{[36]} = -1$ and $\delta_{\text{holo}}^{[36]} = -2$, though).

Furthermore, the results of Ref. [34] were argued to compare favourably with certain characteristics of the energy- and momentum-dependent magnetic susceptibility, as probed by inelastic neutron scattering in LaSrCuO. Specifically, the predicted behaviour of the ratio (here \mathbf{Q} is the antiferromagnetic vector)

$$\left. \frac{\chi_s(\omega, \mathbf{q})}{\omega} \right|_{\omega \rightarrow 0} \sim |\mathbf{q} - \mathbf{Q}|^{\eta} \quad (16)$$

was found to be governed by the exponent $\eta_{\text{holo}} = -10/3$ which is indeed close to the measured $\eta_{\text{exp}} = -3$ [37, 38]. In contrast, the alternate scheme of Ref. [36] yields the exponent $\eta = 3$ the value of which appears to be right on the data.

Also, the momentum integral $T \int d\mathbf{q} \frac{\chi_s(\omega, \mathbf{q})}{\omega} \Big|_{\omega \rightarrow 0}$

turned out to be constant in both schemes of Refs. [34, 36], again in agreement with the data of Refs. [37, 38]. In turn, the uniform magnetic and charge susceptibilities

$$\chi_s = \frac{d^2 f}{dB^2} \sim T^\xi, \quad \chi_c = \frac{d^2 f}{d\mu^2} \sim T^\xi \quad (17)$$

were found to be governed by the exponents

$$\xi_{\text{holo}}^{[34]} = -1.5, \quad \xi_{\text{holo}}^{[36]} = 0.5 \quad (18)$$

and

$$\xi_{\text{holo}}^{[36]} = -2, \quad \xi_{\text{holo}}^{[36]} = 0 \quad (19)$$

in Refs. [34, 36], respectively, thus providing additional means of discriminating between the two scenarios. Notably, in either scheme the Wilson ratio $c/\chi_c T$ conformed to a constant, once again in accord with experiment.

Although a few of the subsequent publications [39, 40] acknowledged the better overall agreement with the predictions of Ref. [36], great many other papers pursued the various versions of the holographic approach, akin to Refs. [33, 34].

To that end, the practical AdS/CMT has been reinventing itself as advanced hydrodynamics of strongly coupled quantum matter. The renewed appreciation for and novel applications of such a well-established field as hydrodynamics (which, while suggesting some formal holographic connections, had long been discussed before the rise of holography) emerged out of the recent experimental discoveries of the electron hydrodynamic regime in mono- and bilayer graphene, (Al, Ga)As heterostructures, PdCoO₂, herbertsmithite, etc. Among other things, it also resulted in a renewed interest in the anomalous transport in the cuprates.

3. (Non)holographic hydrodynamics

The intimate relation between classical gravity and hydrodynamics has long been known as a particu-

lar take on the AdS/CFT referred to as ‘fluid-gravity’ correspondence [26–32]. The crux of the matter lies in the deep similarity between the asymptotic near-boundary behaviour of the Einstein equations for the bulk metric and the Navier–Stokes ones describing a dual boundary fluid in one lesser dimension. Albeit usually truncated and, therefore, approximate such relations can be systematically improved, thus enabling certain computational simplifications.

The magneto-hydrodynamic transport coefficients were first derived in the early work of Ref. [41] under the assumption of a (pseudo) relativistic kinematics of the charge and heat carriers. While the underlying hydrodynamic equations were mimicked after those of a quark–gluon plasma, they would also be considered applicable to the electron transport in graphene (the actual hydrodynamic equations describing monolayer graphene appear to be somewhat different due to the presence of an extra hydrodynamic ‘imbalance’ mode as well as the expressly non-relativistic nature of the Coulomb interaction, though [42–45]).

To the lowest order in the magnetic field the hydrodynamic results of Ref. [41] reduce to the relations

$$\begin{aligned} \sigma &\sim \sigma_{\text{coh}} + \sigma_{\text{in}}, \\ \sigma_{\text{H}} &\sim B\sigma_{\text{coh}}(\sigma_{\text{coh}} + 2\sigma_{\text{in}})n^{-1}, \\ \alpha &\sim s\sigma_{\text{coh}}n^{-1}, \\ \alpha_{\text{H}} &\sim sB\sigma_{\text{coh}}(\sigma_{\text{coh}} + \sigma_{\text{in}})n^{-2}, \\ \kappa &\sim s^2T\sigma_{\text{coh}}n^{-2}, \\ \kappa_{\text{H}} &\sim Bs^2T\sigma_{\text{coh}}^2n^{-3}. \end{aligned} \quad (20)$$

Regarding these expressions the all-time important issue has been that of the intrinsically additive or ‘inverse Matthiessen’ structure of the kinetic coefficients [46, 47].

In particular, the hydrodynamic (as well as the alternate memory-matrix) calculations of the DC conductivity revealed its decomposition onto the generalized coherent (‘Drude’) contribution [48–57]

$$\sigma_{\text{coh}} = \frac{\chi_{JP}^2}{\chi_{PP}\Gamma_{\text{mr}}} \quad (21)$$

and its intrinsic ‘incoherent’ counterpart. In the relativistically invariant holographic context it was estimated as

$$\sigma_{\text{inc}} = \left(\frac{sT}{\epsilon + P} \right)^2, \quad (22)$$

where s and $\epsilon + P$ are entropy and enthalpy densities, respectively [58].

The coherent term (21) is controlled by the momentum relaxation rate Γ_{mr} together with the momentum–momentum χ_{pp} and current–momentum χ_{jp} susceptibilities. The latter vanishes if the operator of electric current is orthogonal to that of momentum.

The coherent term receives contributions from all the sources of momentum relaxation (impurities, phonons, umklapp, boundary scattering, etc.). In turn, the second term provides for a finite conductivity in a neutral relativistic plasma in the absence of any external mechanism of momentum relaxation. Physically, it is due to the momentum-conserving Coulomb drag between the opposite charge carriers.

Similar incoherent terms were argued to appear in the other thermo-electric coefficients, $\alpha_{\text{inc}} = -\mu\sigma_{\text{inc}}/T$ and $\bar{\kappa}_{\text{inc}} = \mu^2\sigma_{\text{inc}}/T$, which, however, cancel against each other in the zero-current coefficient κ . Recently, it was argued that similar terms must be introduced into the Hall components of the kinetic coefficients as well [59].

According to the popular scenario of Ref. [60], in the conjectured quantum-critical regime the incoherent contribution σ_{inc} is supposed to dominate the Ohm conductivity, thus determining the exponent α , while the Hall response would be controlled by σ_{coh} .

Elaborating further on this proposal, in Refs. [61, 62] the coherent and incoherent terms, alongside the carrier density n , were chosen to behave as

$$\begin{aligned} \sigma_{\text{coh}}^{[61,62]} &\sim T^{-2}, \quad \sigma_{\text{inc}}^{[61,62]} \sim T^{-1}, \\ n^{[61,62]} &\sim T^0, \quad s^{[61,62]} \sim T, \end{aligned} \quad (23)$$

as if the fermion system was deep in the degenerate regime and had a well developed Fermi surface with a finite Fermi momentum $\sim n^{0.5}$.

Besides, the scenario of Refs. [61, 62] produced a list of other exponents

$$\begin{aligned} \gamma_{\text{hydro}}^{[61,62]} &= -3, \quad \mu_{\text{hydro}}^{[61,62]} = -1, \\ \lambda_{\text{hydro}}^{[61,62]} &= 1, \quad \rho_{\text{hydro}}^{[61,62]} = 0, \end{aligned} \quad (24)$$

that could be contrasted against the data (3, 4, 7) as well (*spoiler*: with only a limited success).

Also, the assumptions (23) were made in Refs. [63–65] where yet another version of the holographic, the so-called DBI (Dirac–Born–Infeld), approach was utilized, thus resulting in the same ostensible match for the experimental Eqs. (1–3).

However, in reality the desired dependences (23) might be rather difficult to conform to. Specifically, for a temperature-independent density n the low- T behaviour becomes non-relativistic and Eq. (22) yields $\sigma_{\text{inc}} \sim s^2T^2$. Instead of behaving as T^{-1} , as per Eq. (23), σ_{inc} then vanishes with temperature as T^4 and, therefore, could hardly compete with $\sigma_{\text{coh}} \sim 1/\Gamma_{\text{mr}}$. Indeed, the rate Γ_{mr} either remains almost constant (impurity scattering) or even decreases with decreasing T (phonons or Baber umklapp scattering). Either way, the assumed T^{-1} behaviour does not readily occur.

In the opposite, high- T , limit Eq. (22) approaches a temperature-independent constant of order unity (or, rather, e^2/h). This would be typical for, e.g. (pseudo)relativistic 2 + 1-dimensional fermions in mono-layer graphene which are governed by the unscreened 3-dimensional Coulomb interactions.

Interestingly enough, this behaviour would also be shared by the zero-density fermions with a quadratic dispersion, akin to that in (untwisted) bilayer graphene. In the latter case, the density of thermal excitations $n \sim T$ cancels against the inelastic Coulomb scattering rate (10), thus yielding an (approximate) constant [66–71].

However, it should be kept in mind that in the limit of a strongly T -dependent carrier density, the criterion of ‘hydrodynamicity’ ($\Gamma \gg \Gamma_{\text{mr}}$) further decouples from the assumed dominance of σ_{inc} which would be, by and large, controlled by n/Γ with the pertinent inelastic rate given by, e.g. Eq. (9) or (10). As a result, the range of parameters at which hydrodynamics is expected to work might be bounded at both low- and high- T .

Indeed, in the presence of competing sources of momentum relaxation such recognized hydrodynamic systems as monolayer and magic-angle-twisted bi-layer graphene were predicted to manifest their fluid-like behaviour only within a relatively narrow window of temperatures where the disorder

and phonon scattering mechanisms set the lower and upper bounds, respectively. On the other hand, in the untwisted bilayer graphene the hydrodynamic regime is not expected to be bounded from above [66–71].

In that regard, it is instructive to mention the recent work on the compound BSCO with a low $T_c \sim 10$ K [72] which reported the different measured exponents

$$\beta_{\text{exp}}^{[72]} = -1.5, \delta_{\text{exp}}^{[72]} = -3, \mu_{\text{exp}}^{[72]} = -2.5. \quad (25)$$

The new values of δ and μ were extracted from the data taken in the narrow range of temperatures between 20 and 40 or 60 K (above which the Nernst signal changes sign), respectively. For comparison, the results of Ref. [10] for σ_H , κ_H and L_H in underdoped YBCO were collected over a wider range of temperatures, yet the authors refrained from fitting them with any particular power laws.

Nonetheless, the concomitant theoretical analysis of Ref. [72] based on the hydrodynamic Eqs. (20) claimed to have been able to explain all of Eqs. (1, 4, 25) by making the assumptions

$$\begin{aligned} \sigma_{\text{coh}}^{[72]} &\sim T^0, \sigma_{\text{inc}}^{[72]} \sim T^{-1}, \\ n^{[72]} &\sim T^{1.5}, s^{[72]} \sim T. \end{aligned} \quad (26)$$

Similar to the earlier proposals [60–65] electrical transport was still going to be dominated by σ_{inc} . However, this time around it was supposed to occur at *low* rather than *high* temperatures between 10 and 100 K (despite the fact that the must-have exponent (1) could be observed up to 700 K).

Moreover, in Ref. [72] the main source of momentum relaxation was attributed to the non-quasi-particle transport through a charge density wave (CDW) [73–75] and an intricate cancellation between the different T -dependent factors in σ_{coh} was required to achieve (26).

Lastly, by having made the above assumptions, Ref. [72] would also be forced into the less wanted predictions

$$\rho_{\text{hydro}}^{[72]} = -1, \lambda_{\text{hydro}}^{[72]} = -1.5, \quad (27)$$

which are not immediately supported by the data.

In fact, if the agreement asserted in Ref. [72] were indeed there, the assumed behaviour of

the carrier density (26) would have appeared to be in conflict with the underlying assumption of the relativistic (that is, $z = 1$) kinematics of carriers, as well as the conjectured scaling of entropy. Besides, it would also call for the inelastic scattering rate $\Gamma_{\text{in}} \sim n/\sigma_{\text{inc}} \sim T^{2.5}$ for which there seems to be no known microscopic mechanism.

Indeed, in a generic d -dimensional system with the dispersion $\epsilon \sim p^z$ the carrier density scales as $n \sim T^{d/z}$. Thus, taken at its face value Eq. (26) would have implied $z = 4/3$ for $d = 2$. Incidentally, this value of z coincides with that proposed in Ref. [34], despite the fact that instead of the novel Eq. (25) Ref. [34] aimed at reproducing the ‘orthodox’ values of the exponents in Eqs. (1–4, 7).

In light of the lingering tension between the available data and all the aforementioned proposals, it is interesting to point out that the hydrodynamic formulas can still produce the exponents that are equal or close to the observed ones by adopting the scenario of Ref. [36].

Namely, under the minimal assumptions

$$\sigma_{\text{hydro}} \sim T^{-1}, n_{\text{hydro}} \sim T, \quad (28)$$

where no distinction is to be made between the ‘Drude’ and incoherent parts of the conductivity and barring any fine-tuned cancellations *à la* Sondheimer, the hydrodynamic Eqs. (20) yield the following exponents:

$$\begin{aligned} \alpha_{\text{hydro}}^{[36]} &= -1, \beta_{\text{hydro}}^{[36]} = -2, \gamma_{\text{hydro}}^{[36]} = -4, \\ \delta_{\text{hydro}}^{[36]} &= -2, \nu_{\text{hydro}}^{[36]} = 1, \mu_{\text{hydro}}^{[36]} = -2, \\ \lambda_{\text{hydro}}^{[36]} &= 0, \rho_{\text{hydro}}^{[36]} = 0. \end{aligned} \quad (29)$$

The ansatz (28) describes, e.g. a system of non-degenerate two-dimensional fermions with a quadratic dispersion and generic scattering rate (10).

Of course, the simple schemes with T -dependent carrier density and a single scattering rate, including those with $n \sim T$, have been discussed since the early days of the high- T_c era [76, 77]. As regards the cuprates, the underdoped YBaCuO and HgBaCuO show the presence of small electron pockets, in contrast with the large hole-like Fermi surface which develops in the overdoped regime above the critical doping p^* . This is consistent with the reports of a dramatic drop in the low-temperature carrier density (evaluated by the Hall number

n_H) from $n_H \approx 1 + p$ to $n_H \approx p$ upon crossing into the pseudogap phase [10–14].

The generic rate (9) could originate from the Baber mechanism (although the applicability of hydrodynamics would then be rather questionable) whose effectiveness depends on whether or not the quasiparticle dispersion, Fermi surface topology, and spatial dimension conspire to provide for the comparable rates of the normal and umklapp inelastic scattering processes. It is believed, though, that in the cuprates both the multipocketed (in the under- and optimally-doped cases) as well as the extended concave (in the overdoped case) hole Fermi surfaces might comply with the necessary conditions outlined in Refs. [78–80].

The matters become further complicated due to the possible onset of CDW order, as in orthorhombic YBCO and tetragonal HgBaCuO which may induce Fermi-surface reconstruction. However, some authors believe that this might be a secondary phenomenon occurring only in high magnetic fields and at temperatures below the zero-field T_c [25].

By contrast, when coupled with the Fermi liquid-like scattering rate (9) observed across much of the entire cuprates' phase diagram [18–22, 25], the T -dependent carrier density may be hinting at some alternate theoretical scenarios that neither exploit the notion of quantum criticality, nor attribute any special role to the incidental CDW order. In particular, the work of Ref. [25] emphasizes a potential importance of the local (pseudo)gaps produced by some intrinsic microscopic inhomogeneity (whose type of local physics would unlikely be conducive to any holographic speculations).

4. Summary

To summarize, this note exposes the systemic problem inherent to the 'bottom-up' holographic approach which tends to favour technical convenience at the expense of physical insight.

It is, of course, quite possible that the seemingly suggestive scaling exhibited by a variety of the experimental probes is, in fact, only approximate and limited to certain, insufficiently broad, ranges of parameters, thus making it virtually impossible to explain all such findings within the same paradigm.

Moreover, as regards the general holographic task of constructing a comprehensive catalogue of

all the different types of 'strange-metallic' behaviour [26–32], the traditional focus on the cuprates with its prime goal of reproducing the above scaling dependences appears to be much too narrow. To that end, there exists a plethora of other (e.g. heavy-fermion) compounds where the unexplained power-law dependences are abound. As an added challenge, in many cases the apparent exponent z remains finite, thus breaking out of the restrictive confines of the AdS₂ scenario characterized by the diverging z .

Thus, should the true status of the holographic studies be finally ascertained and its machinery proved viable, there would still be a plenty of other condensed matter systems [81] waiting to be tackled by this purportedly promising method for studying strongly correlated systems.

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TVARI HOLOGRAFIŠKOJI FENOMENOLOGIJA KUPRATAMS

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