Lithuanian Journal of Physics, Vol. **48**, No. 1, pp. 49–58 (2008) doi:10.3952/lithjphys.48113

PROPERTIES OF AUGER ELECTRONS FOLLOWING IONIZATION OF POLARIZED ATOMS BY POLARIZED ELECTRONS

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Received 29 November 2007; accepted 22 February 2008

The general expression for the differential cross-section describing Auger decay following the ionization of polarized atoms by polarized electrons is obtained in a two-step approximation. In the case of ionization of non-polarized atoms by non-polarized electrons, the expressions for the parameters of angular distribution of Auger electrons and angular correlations between Auger and escaping electrons are derived as special cases of the general expression. The magnetic dichroism in the angular distribution of Auger electrons as well as in the total cross-section of polarized atoms ionized by non-polarized electrons are also investigated.

Keywords: ionization of atoms by electron impact, polarization, angular distribution, Auger electrons

PACS: 31.15.+p, 34.80.Dp, 32.80.Hd

1. Introduction

Inner-shell ionization of atoms by electrons and electromagnetic radiation is a powerful tool for the investigation of matter and interactions and have both theoretical and practical importance [1]. The ionized atom can 'remember' the direction of polarization of the incident electron or photon and the following Auger electron may have a nonisotropic angular distribution [2]. For the investigation of such processes, a number of methods have been developed [2–7]. Density matrix formalism [3] is the usual method for the investigation of polarization and angular distributions in two-step processes. A unified quantum collision theory in the wave-packet approach for the analysis of the kinematics of various atomic collision processes by using the density matrix and helicity formulations was proposed in [4]. Recently Da Pieve et al. [5] have commenced studying the angular correlation between a photoelectron and a subsequent Auger electron from atomic target by using single-particle approach avoiding density matrix formalism. For the calculation of polarized line emission, the method based on a collisional-radiative kinetic model of magnetic sublevel populations was recently used by Hakel et al. [6]. Another method applied as an alternative approach with respect to the usual density matrix formalism was based on the methods developed in the atomic theory [7–9]. In the latter method, the probability or cross-section of the interaction was expressed as multiple expansion over the multipoles (irreducible tensors) of the state of all particles taking part in the process both in initial and final states. The extension of this method to the multi-step processes was presented in [10]. The ionization of polarized atoms by polarized electrons was investigated by Kupliauskienė and Glemža [11] with the help of methods of the theory of an atom [9] in distorted wave approximation. In the case of ionization of atoms by fast non-polarized electrons a simpler approach like plane wave Born approximation (PWBA) can be applied [12] to describe the angular distribution and spin polarization of a slower emitted electron from polarized and non-polarized atom as well as the alignment of ionized atoms.

Auger decay following the photoionization of polarized atoms was investigated in [13, 14]. The investigations of Auger decay following the ionization of atoms by electrons are not numerous. Angular distribution and polarization of Auger electrons from non-polarized atoms ionized by non-polarized electrons was discussed by Blum et al. [15]. Numerical results for the anisotrophy parameters were presented for cases where the final ion had a non-vanishing total angular momentum. In the case of ionization of non-polarized atoms by non-polarized fast electrons, the expressions for the study

ISSN 1648-8504

of angular correlations between Auger and one of the emitted electrons were obtained by Berezhko et al. [16] by using density matrix approach.

The main task of the present work was the derivation of a general expression for differential cross-section describing Auger decay following the ionization of polarized atoms by polarized electrons in non-relativistic approximation, the special cases of which were suitable for the interpretation of the experimental results. The next section of the present work is devoted to obtaining the general expression. Its special cases are presented in Section 3. The inequality fine structure splitting $\gg$ line width $\gg$ hyperfine structure splitting is also assumed. It ussually holds for the case of inner shell ionization of atoms. Then the ion formed can be specified by the total angular momentum $\mathbf{J}$ of electronic shells. The modifications enabling one to take into account hyperfine structure splitting can be easily made [8, 9].

2. General expression

In two-step approximation, the expression of the fourfold differential cross-section describing Auger decay following the ionization of polarized atoms by polarized electrons

$$\begin{aligned} & e^-(\mathbf{p}_0 m_0) + A(\alpha_0 J_0 M_0) \to A^+(\alpha_1 J_1 M_1) + e^-(\mathbf{p}_1 m_1) + e^-(\mathbf{p}_2 m_2) \\ & \to A^{2+}(\alpha_2 J_2 M_2) + e^-(\mathbf{p}_1 m_1) + e^-(\mathbf{p}_2 m_2) + e^-(\mathbf{p}_\mathrm{A} m_\mathrm{A}) \end{aligned} \tag{1}$$

can be written in the form of expansion over multipoles of the non-registered intermediate state of an ion $A^+(\alpha_1 J_1 M_1)$ by using the method proposed in [10] as follows:

$$\begin{aligned} & \frac{\mathrm{d}^4\sigma(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_1 m_1 \mathbf{p}_2 m_2 \to \alpha_2 J_2 M_2 \mathbf{p}_\mathrm{A} m_\mathrm{A})}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \mathrm{d}\Omega_\mathrm{A}} \\ & = \sum_{K_1 N_1} \frac{\mathrm{d}^3\sigma_{K_1 N_1}(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \mathbf{p}_1 m_1)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2} \, \frac{\mathrm{d}W^\mathrm{A}_{K_1 N_1}(\alpha_1 J_1 \to \alpha_2 J_2 M_2 \mathbf{p}_\mathrm{A} m_\mathrm{A})}{\mathrm{d}\Omega_\mathrm{A}} \, . \end{aligned} \tag{2}$$

In (1), α_i indicates the configuration and other quantum numbers, J_i is the total angular momentum, and M_i is its projection onto the chosen direction, for the atoms (ions) in the initial ($i = 0$), intermediate ($i = 1$), and final ($i = 2$) states; $\mathbf{p}_i$, m_i, and $\mathrm{d}\Omega_i$ describe the momentum of an electron, its projection onto the chosen direction, and scattering angle, respectively, for the projectile ($i = 0$), scattered ($i = 1$), emitted ($i = 2$), and Auger ($i = \mathrm{A}$) electrons, ε_2 is the energy of emitted electron ($p_2 = (2\varepsilon_2)^{1/2}$ in atomic units).

The expression for the first term in (2) can be obtained from Eq. (13) of [11] by applying the procedure described in [9, 10] and it is as follows:

$$\begin{aligned} & \frac{\mathrm{d}^3\sigma_{K_1 N_1}(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \mathbf{p}_1 m_1)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2} = C_2 (4\pi)^{3/2} [2K_1 + 1]^{1/2} \\ & \times \sum_{\substack{K, K_0, K_0', K_{\lambda 0}, K_{s0}, K', \\ K_1', K_2', K_{\lambda 1}, K_{s1}, K_{\lambda 2}, K_{s2}}} \mathcal{B}^\mathrm{ion}(K_0, K_0', K, K_{\lambda 0}, K_{s0}, K_1, K', K_{\lambda 1}, K_{s1}, K_1', K_{\lambda 2}, K_{s2}, K_2') \\ & \times \sum_{\substack{N, N_0, N_0', N_{\lambda 0}, N_{s0}, N', \\ N_1', N_2', N_{\lambda 1}, N_{s1}, N_{\lambda 2}, N_{s2}}} \begin{bmatrix} K_{\lambda 0} \, K_{s0} \, K_0' \\ N_{\lambda 0} \, N_{s0} \, N_0' \end{bmatrix} \begin{bmatrix} K_0 \, K_0' \, K \\ N_0 \, N_0' \, N \end{bmatrix} \begin{bmatrix} K_1 \, K' \, K \\ N_1 \, N' \, N \end{bmatrix} \begin{bmatrix} K_2' \, K_1' \, K' \\ N_2' \, N_1' \, N' \end{bmatrix} \begin{bmatrix} K_{\lambda 1} \, K_{s1} \, K_1' \\ N_{\lambda 1} \, N_{s1} \, N_1' \end{bmatrix} \begin{bmatrix} K_{\lambda 2} \, K_{s2} \, K_2' \\ N_{\lambda 2} \, N_{s2} \, N_2' \end{bmatrix} \\ & \times Y^*_{K_{\lambda 0} N_{\lambda 0}}(\hat{p}_0) \, Y_{K_{\lambda 1} N_{\lambda 1}}(\hat{p}_1) \, Y_{K_{\lambda 2} N_{\lambda 2}}(\hat{p}_2) \\ & \times T^{*K_0}_{N_0}(J_0, J_0, M_0 | \hat{J}_1) \, T^{*K_{s0}}_{N_{s0}}(s, s, m_0 | \hat{s}) \, T^{K_{s1}}_{N_{s1}}(s, s, m_1 | \hat{s}) \, T^{K_{s2}}_{N_{s2}}(s, s, m_2 | \hat{s}) \, . \end{aligned} \tag{3}$$

Here

$$C_2 = \frac{2p_1p_2}{\pi^2 p_0}\,, \quad T_N^K(J,J,M|\hat{J}) = (-1)^{J-M}\left[\frac{4\pi}{2J+1}\right]^{1/2}\begin{bmatrix} J & J & K \\ M & -M & 0\end{bmatrix} Y_{KN}(\hat{J})\,,$$

the hat denotes the polar and azimuthal angles of the orientation with respect to the chosen z axis, and

$$\mathcal{B}^{\mathrm{ion}}(K_0,K_0',K,K_{\lambda 0},K_{s0},K_1,K',K_{\lambda 1},K_{s1},K_1',K_{\lambda 2},K_{s2},K_2')$$
$$= \sum_{\substack{\lambda_0,\lambda_0',\lambda_1,\lambda_1',\lambda_2,\lambda_2',j_0,j_0',\\ j_1,j_1',j_2,j_2',J,J',j,j'}} (2J+1)(2J'+1)(2s+1)(-1)^{\lambda_0'+\lambda_1'+\lambda_2'}\begin{bmatrix}\lambda_0 & \lambda_0' & K_{\lambda 0}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\lambda_1 & \lambda_1' & K_{\lambda 1}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\lambda_2 & \lambda_2' & K_{\lambda 2}\\ 0 & 0 & 0\end{bmatrix}$$
$$\times\langle\alpha_1 J_1,\varepsilon_2\lambda_2(j_2)\varepsilon_1\lambda_1(j_1)(j)J||V||\alpha_0 J_0,\varepsilon_0\lambda_0(j_0)J\rangle\ \langle\alpha_1 J_1,\varepsilon_2\lambda_2'(j_2')\varepsilon_1\lambda_1'(j_1')(j')J'||V||\alpha_0 J_0,\varepsilon_0\lambda_0'(j_0')J'\rangle^*$$
$$\times\big[(2s+1)(2\lambda_0+1)(2\lambda_0'+1)(2\lambda_1+1)(2\lambda_1'+1)(2\lambda_2+1)(2\lambda_2'+1)(2j_0+1)(2j_0'+1)(2j_1+1)(2j_1'+1)$$
$$\times(2j_2+1)(2j_2'+1)(2j+1)(2j'+1)(2J_0+1)(2J_1+1)(2K_0'+1)(2K_1'+1)(2K'+1)(2K_2'+1)\big]^{1/2}$$
$$\times\begin{Bmatrix} J_0 & K_0 & J_0\\ j_0' & K_0' & j_0\\ J' & K & J\end{Bmatrix}\begin{Bmatrix}\lambda_0' & K_{\lambda 0} & \lambda_0\\ s & K_{s0} & s\\ j_0' & K_0' & j_0\end{Bmatrix}\begin{Bmatrix}J_1 & K_1 & J_1\\ j' & K' & j\\ J' & K & J\end{Bmatrix}\begin{Bmatrix}\lambda_1' & K_{\lambda 1} & \lambda_1\\ s & K_{s1} & s\\ j_1' & K_1' & j_1\end{Bmatrix}\begin{Bmatrix}\lambda_2' & K_{\lambda 2} & \lambda_2\\ s & K_{s2} & s\\ j_2' & K_2' & j_2\end{Bmatrix}\begin{Bmatrix}j_2' & K_2' & j_2\\ j_1' & K_1' & j_1\\ j' & K' & j\end{Bmatrix}. \tag{4}$$

In (4), V marks the operator of the electrostatic interaction between atomic and projectile electrons. The reduced matrix element in (4) consists of the direct and exchange terms where the order of the angular momenta of partial waves in each term is different. The recoupling of these angular momenta was applied in order to save the possibility to use the same formula for the kinematic part in the general expression (4). Then a more complicated expression for the reduced matrix element of the electrostatic interaction is obtained:

$$\langle\alpha_1 J_1,\varepsilon_2\lambda_2(j_2)\varepsilon_1\lambda_1(j_1)(j')J||V||\alpha_0 J_0,\varepsilon_0\lambda_0(j_0)J\rangle = [\alpha_0 L_0|\alpha_1(L_1)lL_0][\alpha_0 S_0|\alpha_1(S_1)sS_0]$$
$$\times[(2L_0+1)(2S_0+1)(2J_0+1)(2J_1+1)(2j_0+1)(2j_1+1)(2j_2+1)(2j+1)]^{1/2}$$
$$\times 2(2l+1)(2\lambda_1+1)\sum_{j',k,k_s,k''}(2j'+1)(2k''+1)(-1)^{j'+j_0+j_1+j_2+j+J_1+J}$$
$$\times\begin{Bmatrix}L_1 & S_1 & J_1\\ l & s & j'\\ L_0 & S_0 & J_0\end{Bmatrix}\begin{Bmatrix}\lambda_0 & s & j_0\\ k & k_s & k''\\ \lambda_1 & s & j_1\end{Bmatrix}\begin{Bmatrix}\lambda_2 & s & j_2\\ k & k_s & k''\\ l & s & j'\end{Bmatrix}\begin{Bmatrix}J_0 & j_0 & J\\ j & J_1 & j'\end{Bmatrix}\begin{Bmatrix}j_2 & j_1 & j\\ j_0 & j' & k''\end{Bmatrix}\Big[(l||C^{(k)}||\lambda_2)(\lambda_1||C^{(k)}||\lambda_0)$$
$$\times R_k(nl\varepsilon_2\lambda_2,\varepsilon_1\lambda_1\varepsilon_0\lambda_0)\delta(k_s,0)+\sum_{k'}\begin{Bmatrix}\lambda_0 & k & \lambda_1\\ l & k' & \lambda_2\end{Bmatrix}(l||C^{(k')}||\lambda_1)(\lambda_2||C^{(k')}||\lambda_0)\,R_{k'}(nl\varepsilon_1\lambda_1,\varepsilon_2\lambda_2\varepsilon_0\lambda_0)\Big]. \tag{5}$$

Here $[\alpha_0 L_0|\alpha_1(L_1)lL_0]$ and $[\alpha_0 S_0|\alpha_1(S_1)sS_0]$ are recoupling matrices which represent the contribution from the part of wave functions described by the configuration, other quantum numbers, and the total orbital and spin angular momenta of an atom.

The second term in (2) has the following expression [13, 14]:

$$\frac{\mathrm{d}W^{\mathrm{A}}_{K_1N_1}(\alpha_1 J_1\to\alpha_2 J_2 M_2\mathbf{p}_{\mathrm{A}} m_{\mathrm{A}})}{\mathrm{d}\Omega_{\mathrm{A}}} = \sum_{\substack{K_{\mathrm{A}}',K_2,K_{\lambda\mathrm{A}},K_{s\mathrm{A}},\\ N_{\mathrm{A}}',N_2,N_{\lambda\mathrm{A}},N_{s\mathrm{A}}}}\mathcal{A}^a(K_1,K_2,K_{\lambda\mathrm{A}},K_{s\mathrm{A}},K_{\mathrm{A}}')\,\times$$

$$\times \begin{bmatrix} K_{\lambda\mathrm{A}} & K_{s\mathrm{A}} & K'_{\mathrm{A}} \\ N_{\lambda\mathrm{A}} & N_{s\mathrm{A}} & N'_{\mathrm{A}} \end{bmatrix} \begin{bmatrix} K_2 & K'_{\mathrm{A}} & K_1 \\ N_2 & N'_{\mathrm{A}} & N_1 \end{bmatrix} T^{K_2}_{N_2}(J_2, J_2, M_2|\hat{J}_2)\, T^{K_{s\mathrm{A}}}_{N_{s\mathrm{A}}}(s_{\mathrm{A}}, s_{\mathrm{A}}, m_{\mathrm{A}}|\hat{s}_{\mathrm{A}})\, \sqrt{4\pi}\, Y_{K_{\lambda\mathrm{A}}N_{\lambda\mathrm{A}}}(\theta_{\mathrm{A}}, \phi_{\mathrm{A}})\,, \quad (6)$$

$$\mathcal{A}^a(K_1, K_2, K_{\lambda\mathrm{A}}, K_{s\mathrm{A}}, K'_{\mathrm{A}}) = \frac{1}{2} \sum_{\lambda_{\mathrm{A}}, j_{\mathrm{A}}, \lambda'_{\mathrm{A}}, j'_{\mathrm{A}}} \langle \alpha_2 J_2 \varepsilon \lambda_{\mathrm{A}}(j_{\mathrm{A}}) J_1 || V || \alpha_1 J_1 \rangle \langle \alpha_2 J_2 \varepsilon \lambda'_{\mathrm{A}}(j'_{\mathrm{A}}) J_1 || V || \alpha_1 J_1 \rangle^*$$

$$\times \left[(2\lambda_{\mathrm{A}}+1)(2\lambda'_{\mathrm{A}}+1)(2J_1+1)(2j_{\mathrm{A}}+1)(2j'_{\mathrm{A}}+1)(2J_2+1)(2s+1)(2K'_{\mathrm{A}}+1)\right]^{1/2}$$

$$\times \begin{Bmatrix} J_2 & j'_{\mathrm{A}} & J_1 \\ K_2 & K'_{\mathrm{A}} & K_1 \\ J_2 & j_{\mathrm{A}} & J_1 \end{Bmatrix} \begin{Bmatrix} \lambda'_{\mathrm{A}} & s & j'_{\mathrm{A}} \\ K_{\lambda\mathrm{A}} & K_{s\mathrm{A}} & K'_{\mathrm{A}} \\ \lambda_{\mathrm{A}} & s & j_{\mathrm{A}} \end{Bmatrix} (-1)^{\lambda'_{\mathrm{A}}} \begin{bmatrix} \lambda_{\mathrm{A}} & \lambda'_{\mathrm{A}} & K_{\lambda\mathrm{A}} \\ 0 & 0 & 0 \end{bmatrix}. \quad (7)$$

Here V is the operator of the electrostatic interaction between Auger electron and electrons in the ion A^{2+}.

The expressions (2), (3), and (6) represent the most general case of the cross-section describing the polarization of all particles participating in the process (1) and the angular distributions as well as angular correlations of all three electrons in the final state. These general expressions can be used to derive the expressions applicable for the specific experimental conditions with smaller number of polarization states specified. To obtain the special cases, one needs to integrate over the angles of one or both escaping and Auger electrons and to sum over the magnetic components of some angular momenta. The following summation and integration formulae from [8] make this easy to do:

$$\sum_M T^K_N(J, J, M|\hat{J}) = \delta(K, 0)\, \delta(N, 0)\,, \quad (8)$$

$$\int_0^\pi \int_0^{2\pi} Y_{KN}(\theta, \phi) \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi = \sqrt{4\pi}\, \delta(K, 0)\, \delta(N, 0)\,. \quad (9)$$

Some more simple examples are presented in the work as demonstrations. They cover the most frequently measured set of characteristics and used experimental geometries.

3. Special cases

3.1. Total cross-section for the Auger decay following the ionization of non-polarized atoms by non-polarized electrons

The total cross-section describing the Auger decay following the ionization of non-polarized atoms by non-polarized electrons can be obtained from the general expression (2) by summation over the magnetic components of the particles in the final state and averaging over them in the initial state as well as integration over the angles of scattered, emitted, and Auger electrons. The result of these procedures can be written as follows:

$$\frac{\mathrm{d}\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2 \varepsilon_2 \varepsilon_{\mathrm{A}})}{\mathrm{d}\varepsilon_2} = \frac{1}{2(2J_0+1)}$$

$$\times \sum_{M_0, m_0, M_1, m_1, M_2, m_2, m_{\mathrm{A}}} \int \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \mathrm{d}\Omega_{\mathrm{A}} \frac{\mathrm{d}^4\sigma(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \mathbf{p}_1 m_1 \to \alpha_2 J_2 M_2 \mathbf{p}_{\mathrm{A}} m_{\mathrm{A}})}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \mathrm{d}\Omega_{\mathrm{A}}}$$

$$= W^{\mathrm{A}}(\alpha_1 J_1 \to \alpha_2 J_2) \frac{\mathrm{d}\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \varepsilon_2)}{\mathrm{d}\varepsilon_2}\,. \quad (10)$$

Here ε_0, ε_2, and ε_{A} are the energies (related to momentum as $\varepsilon = p^2/2$) of the projectile, emitted, and Auger electrons, respectively. The energy of the scattered electron ε_1 can be obtained from the energy conservation $\varepsilon_1 = \varepsilon_0 - \varepsilon_2 - \Delta E$, where ΔE is the ionization energy of an atom.

In (10), the first term is the autoionization probability in atomic units defined as

$$W^{\mathrm{A}}(\alpha_1 J_1 \to \alpha_2 J_2) = 2\pi \sum_{\lambda_{\mathrm{A}}, j_{\mathrm{A}}} |\langle \alpha_2 J_2 \varepsilon_{\mathrm{A}} \lambda_{\mathrm{A}}(j_{\mathrm{A}}) J_1 || V || \alpha_1 J_1 \rangle|^2 \,. \tag{11}$$

The second term in (10) is the differential electron-impact ionization cross-section

$$\frac{\mathrm{d}\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \varepsilon_2)}{\mathrm{d}\varepsilon_2} = \frac{8}{\varepsilon_0(2J_0+1)} \mathcal{B}^{\mathrm{ion}}(0,0,0,0,0,0,0,0,0,0,0,0,0) \,, \tag{12}$$

where

$$\mathcal{B}^{\mathrm{ion}}(0,0,0,0,0,0,0,0,0,0,0,0,0) =$$
$$\sum_{\lambda_0,\lambda_1,\lambda_2,j_0,j_1,j_2,j,J} (2J+1) \langle \alpha_1 J_1, \varepsilon_2 \lambda_2(j_2) \varepsilon_1 \lambda_1(j_1) j, J || V || \alpha_0 J_0, \varepsilon_0 \lambda_0(j_0) J \rangle^2 \,. \tag{13}$$

The expression for the total ionization cross-section can be obtained by the integration of (12) over the energies of emitted electrons ε_2 and is as follows:

$$\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1) = \frac{8}{\varepsilon_0(2J_0+1)} \int \mathrm{d}\varepsilon_2 \, \mathcal{B}^{\mathrm{ion}}(0,0,0,0,0,0,0,0,0,0,0,0,0) \,. \tag{14}$$

3.2. Angular distribution of Auger electrons following the ionization of non-polarized atoms by non-polarized electrons

The parameters describing the asymmetry of angular distribution of Auger electrons following the ionization of non-polarized atoms by non-polarized electrons can be obtained from the general expression (2) by summation over the magnetic components of particles in the final state and averaging over them in the initial state, as well as integration over the angles of scattered and emitted electrons. The differential cross-section for the asymmetry of angular distribution of Auger electrons is as follows:

$$\frac{\mathrm{d}\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2 \mathbf{p}_{\mathrm{A}})}{\mathrm{d}\Omega_{\mathrm{A}}} = \frac{1}{2(2J_0+1)}$$
$$\times \sum_{M_0,m_0,M_1,m_1,M_2,m_2,m_{\mathrm{A}}} \int \mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \frac{\mathrm{d}^4\sigma(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \mathbf{p}_1 m_1 \to \alpha_2 J_2 M_2 \mathbf{p}_{\mathrm{A}} m_{\mathrm{A}})}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \mathrm{d}\varepsilon_{\mathrm{A}}}$$
$$= \sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2) \left[1 + \sum_{K>0} \beta_K P_K(\cos\theta) \right] . \tag{15}$$

Here the angle θ is measured from the direction of the projectile electron, $P_K(\cos\theta)$ is the Legendre polynomial, $\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2)$ is the total cross-section of the process (1), i. e. the product of the Auger probability (11) and the total ionization cross-section (14):

$$\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2) = \sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1) \, W^{\mathrm{A}}(\alpha_1 J_1 \to \alpha_2 J_2) \,, \tag{16}$$

and

$$\beta_K = A_K \alpha_K^a \tag{17}$$

is the asymmetry parameter of the angular distribution of Auger electrons. In (17), A_K is the alignment parameter of atoms ionized by electron impact. It has the following expression:

$$A_K = \frac{(2K+1) \int \mathrm{d}\varepsilon_2 \mathcal{B}^{\mathrm{ion}}(0,K,K,K,0,K,0,0,0,0,0,0,0)}{\int \mathrm{d}\varepsilon_2 \mathcal{B}^{\mathrm{ion}}(0,0,0,0,0,0,0,0,0,0,0,0,0)} \,. \tag{18}$$

The second term on the right-hand side in (17) describes the asymmetry in the angular distribution of Auger electrons from an ionized atom,

$$\alpha^a_K = \frac{(2K+1)\,\mathcal{A}^a(K,0,K,0,K)}{\mathcal{A}^a(0,0,0,0,0)}\,. \tag{19}$$

The maximum value of K is determined by the values of total angular momentum J_1 of the intermediate state of an ion and orbital momentum of the emitted Auger electron $K_{\max} = 2\min(J_1, \lambda_A)$. K assumes only even values as the parity is conserved in the Auger decay.

3.3. Angular correlations between Auger and escaping electrons following the ionization of non-polarized atoms by non-polarized electrons

The study of the angular distribution of reaction products by the coincidence method enables one to obtain complex amplitudes of different magnetic sublevels of a system, i. e. it yields more fundamental and more comprehensive information than can be extracted from measurements of the total and even the differential cross-section. The angular correlations between Auger and one of the escaping electrons is measured keeping fixed the direction of one of these electrons and changing the direction of the other. The cross-section of the processes (1) should contain the angles of both electrons. It can be derived from (2) by summation over magnetic quantum numbers of all angular momenta, averaging over the initial states, and integration over the angles of the scattered electron:

$$\frac{\mathrm{d}^3\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \mathbf{p}_2 \to \alpha_2 J_2 \mathbf{p}_A)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_2 \mathrm{d}\Omega_A} = \frac{1}{2(2J_0+1)}$$

$$\times \sum_{M_0, m_0, m_1, M_2, m_2, m_A} \int d\Omega_1 \frac{\mathrm{d}^4\sigma(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \mathbf{p}_1 m_1 \to \alpha_2 J_2 M_2 \mathbf{p}_A m_A)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 d\Omega_2}$$

$$= \sum_{K_1, N_1} \frac{\mathrm{d}^2\sigma_{K_1 N_1}(\alpha_0 J_0 \mathbf{p}_0 \to \alpha_1 J_1 \mathbf{p}_2)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_2} \; \frac{\mathrm{d}W^A_{K_1, N_1}(\alpha_1 J_1 \to \alpha_2 J_2 \mathbf{p}_A)}{\mathrm{d}\Omega_A}\,. \tag{20}$$

Here

$$\frac{\mathrm{d}^2\sigma_{K_1 N_1}(\alpha_0 J_0 \mathbf{p}_0 \to \alpha_1 J_1 \mathbf{p}_2)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_2} = \frac{(4\pi)^2 C_2}{2(2J_0+1)} [2K_1+1]^{1/2} \sum_{K_{\lambda 0}, K_{\lambda 2}} B_1(K_1, K_{\lambda 0}, K_{\lambda 2})$$

$$\times \sum_{N_{\lambda 0}, N_{\lambda 2}} \begin{bmatrix} K_1 & K_{\lambda 2} & K_{\lambda 0} \\ N_1 & N_{\lambda 2} & N_{\lambda 0} \end{bmatrix} Y^*_{K_{\lambda 0} N_{\lambda 0}}(\hat{p}_0)\, Y_{K_{\lambda 2} N_{\lambda 2}}(\hat{p}_2)\,, \tag{21}$$

$$B_1(K_1, K_{\lambda 0}, K_{\lambda 2}) = \mathcal{B}^{\mathrm{ion}}(0, K_{\lambda 0}, K_{\lambda 0}, K_{\lambda 0}, 0, K_1, K_{\lambda 2}, 0, 0, 0, K_{\lambda 2}, 0, K_{\lambda 2})\,. \tag{22}$$

The expression (21) becomes simpler when the direction of the projectile electron coincides with the laboratory z axis. Then $N_{\lambda 0} = 0$, $N_1 = N_{\lambda 2}$, and

$$\frac{\mathrm{d}^2\sigma_{K_1 N_1}(\alpha_0 J_0 p_0 \to \alpha_1 J_1 \mathbf{p}_2)}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_2} = \frac{(4\pi)^{3/2} C_2}{2(2J_0+1)} \sum_{K_{\lambda 0}, K_{\lambda 2}} [(2K_1+1)(2K_{\lambda 0}+1)]^{1/2} \begin{bmatrix} K_1 & K_{\lambda 2} & K_{\lambda 0} \\ N_1 & -N_1 & 0 \end{bmatrix}$$

$$\times B_1(K_1, K_{\lambda 0}, K_{\lambda 2})\, Y_{K_{\lambda 2} - N_1}(\hat{p}_2)\,. \tag{23}$$

The differential Auger probability in (20) has the following expression:

$$\frac{\mathrm{d}W^A_{K_1 N_1}(\alpha_1 J_1 \to \alpha_2 J_2 \mathbf{p}_A)}{\mathrm{d}\Omega_A} = \mathcal{A}^a(K_1, 0, K_1, 0, K_1)\sqrt{4\pi}\, Y_{K_1 N_1}(\theta_A, \phi_A)\,. \tag{24}$$

The expression (20) can be written in the form similar to that by Berezhko et al. [16] and more convenient for analysis of specific coinsidence experiments:

$$\frac{\mathrm{d}^3\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \mathbf{p}_2 \to \alpha_2 J_2 \mathbf{p}_\mathrm{A})}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_2 \mathrm{d}\Omega_\mathrm{A}} = \frac{1}{4\pi} \frac{\mathrm{d}\sigma(\alpha_0 J_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2)}{\mathrm{d}\varepsilon_2}$$

$$\times \left[1 + \sum_{K_1} \alpha^a_{K_1} \sum_{N_1} A_{K_1 N_1}(\theta_2, \phi_2) \left[\frac{4\pi}{2K_1 + 1} \right]^{1/2} Y_{K_1 N_1}(\theta_\mathrm{A}, \phi_\mathrm{A}) \right] . \tag{25}$$

Here

$$A_{K_1 N_1}(\theta_2, \phi_2) =$$

$$\frac{\sum\limits_{K_{\lambda 0}, K_{\lambda 2}} (2K_1 + 1)[2K_{\lambda 0} + 1]^{1/2} B_1(K_1, K_{\lambda 0}, K_{\lambda 2}) \begin{bmatrix} K_1 & K_{\lambda 2} & K_{\lambda 0} \\ N_1 & -N_1 & 0 \end{bmatrix} Y_{K_{\lambda 2} - N_1}(\theta_2, \phi_2)}{B_1(0,0,0)} \tag{26}$$

is the alignment parameter of the ionized atom depending on the angles θ_2, ϕ_2 of the emitted electron. The relation

$$A_{K_1 N_1}(\theta_2, \phi_2) = (-1)^{K_1 - N_1} A_{K_1 - N_1}(\theta_2, \phi_2)$$

reduces the number of independent tensors $A_{K_1 N_1}(\theta_2, \phi_2)$.

3.4. *Magnetic dichroism in the angular distribution of Auger electrons following the ionization of polarized atoms by non-polarized electrons*

Magnetic dichroism is known as the dependence of the intensity of Auger electrons or fluorescence radiation following excitation, ionization, or photoionization of atoms on the direction of the initial polarization of atoms. It provides information about the dynamics of the Auger process complementary to that obtained from the spectra of energy and angular distributions of Auger electrons. In the case of the ionization of polarized atoms by non-polarized electrons, the cross-section for the process (1) suitable to describe the magnetic dichroism can be readily obtained by performing summation of the general expression (2) over the magnetic components of the non-registered angular momenta $M_2, m_1, m_2, m_\mathrm{A}$, averaging over m_0, and integration over the angles of the scattered and emitted electrons:

$$\frac{\mathrm{d}\sigma(\alpha_0 J_0 M_0 \varepsilon_0 \to \alpha_1 J_1 \varepsilon_2 \to \alpha_2 J_2 \mathbf{p}_\mathrm{A})}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_\mathrm{A}} =$$

$$\frac{1}{2} \sum_{m_0, m_1, M_2, m_2, m_\mathrm{A}} \int \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \frac{\mathrm{d}^4\sigma(\alpha_0 J_0 M_0 \mathbf{p}_0 m_0 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \mathbf{p}_1 m_1 \to \alpha_2 J_2 M_2 \mathbf{p}_\mathrm{A} m_\mathrm{A})}{\mathrm{d}\varepsilon_2 \mathrm{d}\Omega_1 \mathrm{d}\Omega_2 \mathrm{d}\varepsilon_2}$$

$$= \sum_{K_1, N_1} \frac{\mathrm{d}\sigma_{K_1 N_1}(\alpha_0 J_0 M_0 \mathbf{p}_0 \to \alpha_1 J_1 \varepsilon_2)}{\mathrm{d}\varepsilon_2} \frac{\mathrm{d}W^\mathrm{A}_{K_1 N_1}(\alpha_1 J_1 \to \alpha_2 J_2 \mathbf{p}_\mathrm{A})}{\mathrm{d}\Omega_\mathrm{A}} . \tag{27}$$

Here

$$\frac{\mathrm{d}\sigma_{K_1 N_1}(\alpha_0 J_0 M_0 \mathbf{p}_0 \to \alpha_1 J_1 \varepsilon_2)}{\mathrm{d}\varepsilon_2} = \frac{C_2}{2} [(4\pi)^5 (2K_1 + 1)]^{1/2} \sum_{K_0, K_{\lambda 0}, N_0, N_{\lambda 0}} \begin{bmatrix} K_0 & K_{\lambda 0} & K_1 \\ N_0 & N_{\lambda 0} & N_1 \end{bmatrix}$$

$$\times B_2(K_0, K_1, K_{\lambda 1}) \, Y^*_{K_{\lambda 0} N_{\lambda 0}}(\hat{p}_0) \, T^{*K_0}_{N_0}(J_0, J_0, M_0 | \hat{J}_0) , \tag{28}$$

where

$$B_2(K_0, K_1, K_{\lambda 1}) = \mathcal{B}^\mathrm{ion}(K_0, K_{\lambda 0}, K_1, K_{\lambda 0}, 0, K_1, 0, 0, 0, 0, 0, 0, 0) . \tag{29}$$

The choice of laboratory z axis along the direction of the projectile electron makes the expression (28) simpler, as $N_{\lambda 0} = 0$ and $N_0 = N_1$:

$$\frac{\mathrm{d}\sigma_{K_1N_1}(\alpha_0 J_0 M_0 \varepsilon_0 \to \alpha_1 J_1 \varepsilon_2)}{\mathrm{d}\varepsilon_2} = (4\pi)^2$$

$$\times \sum_{K_0,N_1} \frac{C_2}{2}[(2K_1+1)(2K_{\lambda 0}+1)]^{1/2} \begin{bmatrix} K_0 & K_{\lambda 0} & K_1 \\ N_1 & 0 & N_1 \end{bmatrix}$$

$$\times B_2(K_0, K_1, K_{\lambda 0})(-1)^{J_0-M_0} \left[\frac{4\pi}{2J_0+1}\right]^{1/2}$$

$$\times \begin{bmatrix} J_0 & J_0 & K_0 \\ M_0 & -M_0 & 0 \end{bmatrix} Y^*_{K_0N_1}(\hat{J}_0)\,. \tag{30}$$

The magnetic dichroism equals to the difference between the cross-sections (30) measured for the opposite directions of the total angular momentum $\mathbf{J}_0$. To obtain its expression one needs to multiply (30) by 2 and to take into account only the terms with odd values of K_0. The odd values of K_0 give contribution due to the properties of the Clebsch–Gordan coefficients [17] in (30).

3.5. Magnetic dichroism in the total cross-section of Auger electrons following the ionization of polarized atoms by non-polarized electrons

The magnetic dichroism reveals itself in the total cross-section of Auger electrons following the ionization of polarized atoms by non-polarized electrons as well. The expression for this cross-section can be obtained by integrating (30) over the angles of the Auger electrons and it is as follows:

$$\frac{\mathrm{d}\sigma(\alpha_0 J_0 M_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2)}{\mathrm{d}\varepsilon_2} = \tag{31}$$

$$W^{\mathrm{A}}(\alpha_1 J_1 \to \alpha_2 J_2) \sum_{K_0} \frac{\mathrm{d}\sigma_{K_0}(\alpha_0 J_0 M_0 \varepsilon_0 \to \alpha_1 J_1)}{\mathrm{d}\varepsilon_2}\,.$$

Here

$$\frac{\mathrm{d}\sigma_{K_0}(\alpha_0 J_0 M_0 \varepsilon_0 \to \alpha_1 J_1)}{\mathrm{d}\varepsilon_2} = \frac{C_2}{2}\left[\frac{2K_0+1}{2J_0+1}\right]^{1/2}$$

$$\times (2K_0+1) B_2(K_0, 0, K_0)(-1)^{J_0-M_0}$$

$$\begin{bmatrix} J_0 & J_0 & K_0 \\ M_0 & -M_0 & 0 \end{bmatrix} P_{K_0}(\cos\theta_0)\,, \tag{32}$$

where θ_0 is the angle between the orientation of the axis to which the projection of the total angular momentum of an atom $\mathbf{J}_0$ is defined and the direction of the projectile electron.

In this case, the ratio

$$\Delta = \frac{\sigma(J_0 M_0) - \sigma(J_0\ -M_0)}{\sigma(J_0 M_0) + \sigma(J_0\ -M_0)} =$$

$$\left\{ \sum_{K_0=\mathrm{odd}} (2K_0+1)^{3/2} B_2(K_0,0,K_0)(-1)^{J_0-M_0} \right.$$

$$\left. \times \begin{bmatrix} J_0 & J_0 & K_0 \\ M_0 & -M_0 & 0 \end{bmatrix} P_{K_0}(\cos\theta_0) \right\}$$

$$\times \left\{ \sum_{K_0=\mathrm{even}} (2K_0+1)^{3/2} B_2(K_0,0,K_0)(-1)^{J_0-M_0} \right.$$

$$\left. \times \begin{bmatrix} J_0 & J_0 & K_0 \\ M_0 & -M_0 & 0 \end{bmatrix} P_{K_0}(\cos\theta_0) \right\}^{-1} \tag{33}$$

is independent of the parameters of the Auger decay. In (33), the definition $\sigma(J_0M_0) = \mathrm{d}\sigma(\alpha_0 J_0 M_0 \varepsilon_0 \to \alpha_1 J_1 \to \alpha_2 J_2)/\mathrm{d}\varepsilon_2$ is used.

In the case of the orientation of $\mathbf{J}_0$ along the direction of incoming electron, θ_0 is 0, and so

$$\Delta = \tag{34}$$

$$\frac{\sum\limits_{K_0=\mathrm{odd}} (2K_0+1)^{3/2}\, B_2(K_0,0,K_0) \begin{bmatrix} J_0 & J_0 & K_0 \\ J_0 & -J_0 & 0 \end{bmatrix}}{\sum\limits_{K_0=\mathrm{even}} (2K_0+1)^{3/2}\, B_2(K_0,0,K_0) \begin{bmatrix} J_0 & J_0 & K_0 \\ J_0 & -J_0 & 0 \end{bmatrix}}\,,$$

where $K_0 \le 2J_0$. Thus, the ratio Δ can be non-zero in the case of $J_0 = 1/2$:

$$\Delta = \frac{(2J_0+1)^{1/2}\, 3\sqrt{3}\, B_2(1,0,1)}{\sqrt{2}\, B_2(0,0,0)}\,. \tag{35}$$

For $J_0 = 1$,

$$\Delta = \frac{(2J_0+1)^{1/2}\, 3\sqrt{3}\, B_2(1,0,1)}{\sqrt{2}\,[B_2(0,0,0) + 5\sqrt{5}\,(2J_0+1)\, B_2(2,0,2)]}\,. \tag{36}$$

4. Concluding remarks

The general expression for the cross-section descibing the Auger decay following the ionization of polarized atoms by polarized electrons is obtained when the two-step approximation can be applied. The expression describes the polarization states and angular distributions of all particles both in the initial and final states. Some special cases suitable for the specific conditions are studied as simpler cases of the general expression. These cases are: the angular distribution of Auger electrons following the ionization of non-polarized atoms by non-polarized electrons, angular correlations between Auger and one of the emitted electrons following the ionization of non-polarized atoms by non-polarized electrons, magnetic dichroism in the angular distribution, and the total cross-section of Auger electrons following the ionization of polarized atoms by non-polarized electrons. For other experimental conditions, the expressions can be easily obtained by using the general expression as well.

Acknowledgements

The study was partially funded by the Joint Taiwan–Baltic Research project and by the Ministry of Education and Science of Lithuania under Contract No. SUT-683.

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AUGER ELEKTRONŲ IŠ POLIARIZUOTAIS ELEKTRONAIS JONIZUOTŲ POLIARIZUOTŲ ATOMŲ SAVYBĖS

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Santrauka

Atomų vidinių sluoksnių jonizacija elektronais yra galingas instrumentas mikropasaulio sandarai ir jame egzistuojančioms sąveikoms tirti. Daug svarbios informacijos galima gauti nagrinėjant poliarizuotas būsenas, kurioms ir yra skirtas šis darbas. Nagrinėjamas dviejų tarpsnių vyksmas, kai elektronais jonizuoti atomai išspinduliuoja Auger elektroną. Gautos patį bendriausią visų dalelių poliarizacijos atvejį aprašančios formulės. Jos yra sudėtingos ir atitinka šiuolaikinėmis sąlygomis sunkiai realizuojamą eksperimentą. Pastarajame dažniausiai ne visos dalelės registruojamos, todėl matuojamos sąsajos yra kur kas paprastesnės. Dėl šios priežasties minėtosioms formulėms suteiktas pavidalas, kuriame realūs atvejai aprašomi tomis pačiomis išraiškomis, prilyginus nuliui rangus, atitinkančius neregistruojamas daleles. Toks pavidalas yra patogus, nes leidžia panaudoti tą pačią programą skirtingiems vyksmams nagrinėti. Darbe aptarti keli tokio nagrinėjimo pavyzdžiai.

Kai visi vyksmo dalyviai yra nepoliarizuoti, turime išskirtinį atvejį. Tuomet formulėse visi poliarizaciją nusakantys rangai lygūs nuliui ir daliniai skerspjūviai sutampa su pilnutiniais. Be to, aptarta kampinė koreliacija tarp pradinio ir Auger bei tarp atplėštojo ir Auger elektronų. Taip pat nagrinėjamas Auger elektronų kampinio pasiskirstymo magnetinis dichroizmas bei pats magnetinis dichroizmas, kuris aprašomas suintegravus ankstesniojo atvejo formules pagal Auger elektrono skriejimo kryptis.