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GENERATION-RECOMBINATION NOISE AND OTHER FEATURES OF DOPED SILICON IN A WIDE TEMPERATURE RANGE

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The characteristics of the generation-recombination (g-r) process in silicon are investigated in a temperature range from 25 to 360 K. In the case of shallow donors, it is shown that the free electron density strongly depends on temperature: only 20% of donors are ionized at shallow donor densities of about 10^{17} cm^{-3} at liquid nitrogen temperature. The maximum of the variance of generation-recombination noise due to the free electron density fluctuations for a silicon sample with shallow donors strongly increases with donor density and shifts with temperature. It is demonstrated that the relative variance of free electron number fluctuations is always equal to 0.5 at low temperatures. The normalized generation-recombination noise spectra are depicted in a very wide frequency range. There is also a detailed investigation of the generation-recombination noise characteristics of an acceptor-partially compensated silicon sample with two donor levels. In this work, the main focus is on the characteristics of silicon doped by shallow donors as it is extremely widely used.

Keywords: free electron density fluctuations, generation-recombination noise, shallow donors, silicon

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1. Introduction

Although the processes of free charge carrier generation (emission) and recombination (capture) have been investigated over the last seventy years [1–3], many experiments have been performed with various semiconductor materials [4–7] and devices [8–15], and explanations are provided. During the generation-recombination (g-r) process, the free charge carrier number fluctuations keep the charge neutrality in the total sample. This term is usually used for describing the free charge carrier number fluctuations caused by donor or acceptor levels. The free charge carrier number fluctuations in equilibrium also produce fluctuations in the resistance of the sample. These resistance fluctuations can be simply measured by using direct current because they produce the flowing current fluctuations which are proportional to the square of the magnitude of the direct current. In the case of the deep defect levels, the generation-recombination process is usually described as a charge carrier emission and capture, or retrapping process. In the latter case, the free charge carrier fluctuations are caused by localized defect states with different energy levels in the band gap and are characterized by various free carrier relaxation times [16]. The free charge carrier emission-capture process is also thermally activated and does not change the neutrality condition in the sample. According to Refs. [6, 17], the relaxation times of the g-r process for silicon monocrystals are distributed in the time interval from 0.1 s to 10 μs. In Ref. [18], it is demonstrated that the charge carrier capture-emission process is the main source of the low-frequency noise in homogeneous semiconductors and this process can produce the free charge carrier mobility fluctuations in particular cases [19].

In this work, important properties of the generation-recombination process in a silicon sample doped with shallow donors will be presented.

The main focus here will be on the temperature dependence of the generation-recombination noise.

2. Temperature dependence of the generation-recombination noise

2.1. Silicon sample with one partially ionized donor level

Due to the effect of the thermal lattice vibration in donor-type semiconductors some electrons are excited from donor energy levels into the conduction band (generation process), and at the same time there happens the reverse process (recombination): some electrons are captured by ionized donors. To estimate the characteristics of the g-r noise, it is sufficient to find the generation $g(n)$ and recombination $r(n)$ rate expressions from the physical model.

Consider the g-r noise temperature properties for a donor-type silicon sample with the volume $V = 0.01$ cm^3 and the donor density $n_\mathrm{d} = 10^{15}$ cm^{-3} at the energy level $E_\mathrm{d} = 0.1$ eV below the conduction band (Fig. 1). In the general case, the average free electron density n_0 is described by the following equation [20]:

$$n_0 = n_\mathrm{d} - n_\mathrm{d}^0 = n_\mathrm{d}\left[1 - \frac{1}{1+\beta\exp(-\varepsilon_\mathrm{d} - \eta)}\right]$$

$$= \frac{n_\mathrm{d}}{1+\beta^{-1}\exp(\varepsilon_\mathrm{d}+\eta)} = N_\mathrm{c} F_{1/2}(\eta), \tag{1}$$

where $\varepsilon_\mathrm{d} = E_\mathrm{d}/kT$, $\eta = E_\mathrm{F}/kT$ (here E_F is the Fermi level energy), and n_d^0 is the neutral donor density, for donors $\beta^{-1} = 2$.

$$N_\mathrm{c} = 2(2\pi mkT/h^2)^{3/2}, \tag{2}$$

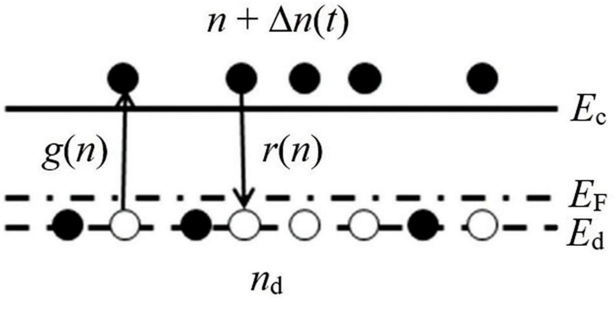

Fig. 1. Illustration of generation and recombination processes in a donor-type semiconductor with partially ionized donors.

$$F_{1/2}(\eta) = \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\varepsilon^{1/2}\mathrm{d}\varepsilon}{1+\exp(\varepsilon-\eta)}. \tag{3}$$

For silicon $N_\mathrm{c} = 2.746\cdot 10^{19}(T/295)^{3/2}$ cm^{-3}.

Dependences of the free electron density n_0 and the Fermi energy E_F on temperature for silicon with $n_\mathrm{d} = 10^{15}$ cm^{-3} at the energy level $E_\mathrm{d} = 0.1$ eV are presented in Fig. 2. When the Fermi energy level coincides with the donor energy level, the free electron density in the conduction band is $n_0 = n_\mathrm{d}/3$. As Fig. 2 shows, all donors are ionized at temperature $T > 200$ K. In the very low temperature range, Eq. (1) can be approximated as

$$n_0 \approx (\beta n_\mathrm{d} N_\mathrm{c})^{1/2}\exp(-E_\mathrm{d}/2kT). \tag{4}$$

The rate of free electron generation $g(n)$ is proportional to the non-ionized donor density $n_\mathrm{d} - n_\mathrm{d}^+$, i.e. to the density of electrons in the donor energy level E_d:

$$g(n) = a(n_\mathrm{d} - n_\mathrm{d}^+) = a(n_\mathrm{d} - n). \tag{5}$$

Here a is the proportionality coefficient, and $n = n_0 + \Delta n$ is a fluctuating quantity. And the rate of electron recombination $r(n)$ is proportional to the density of the free electrons and to the empty (ionized) donor centre density $n_\mathrm{d}^+ = n$:

$$r(n) = \gamma n \cdot n_\mathrm{d}^+ = \gamma n^2. \tag{6}$$

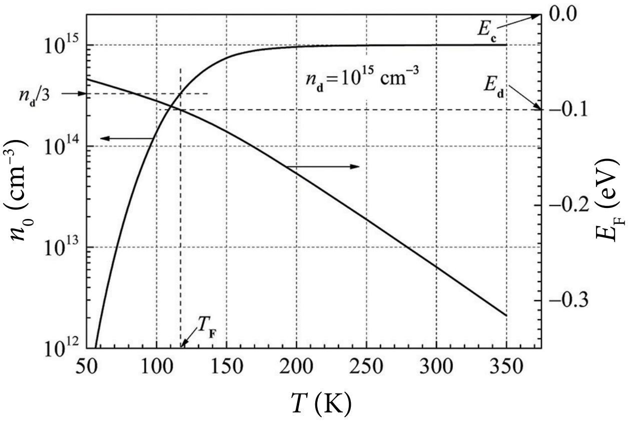

Fig. 2. Dependences of the free electron density n_0 (left scale) and the Fermi energy E_F (right scale) on temperature for silicon with the donor density $n_\mathrm{d} = 10^{15}$ cm^{-3} at the energy level $E_\mathrm{d} = 0.1$ eV (here T_F is the temperature when the Fermi level energy coincides with the donor level energy).

Here γ is the recombination coefficient: $\gamma = v_T\sigma_s$, v_T is the electron thermal velocity, and σ_s is the electron capture cross-section. The coefficient a can be found from the equilibrium condition $g(n_0) = r(n_0)$:

$$a = \gamma \frac{n_0^2}{n_\mathrm{d} - n_0}. \tag{7}$$

The relaxation time of free electrons is estimated in the following way [2],

$$\tau_0 = \frac{1}{r'(n_0) - g'(n_0)} = \frac{1}{2\gamma n_0 + a} = \frac{n_\mathrm{d} - n_0}{\gamma n_0 (2n_\mathrm{d} - n_0)}, \tag{8}$$

and the variance of the free electron density fluctuations is calculated as

$$\left\langle \Delta n^2 \right\rangle = g(n_0)\tau_0 / V = \frac{n_0(n_\mathrm{d} - n_0)}{(2n_\mathrm{d} - n_0)V}, \tag{9}$$

where $V = 0.01$ cm^3. Including the fact that relaxation time is approximately inversely proportional to n_0, and $\gamma \sim T^{1/2}$, dependences of the relaxation time τ_0 and τ_0 multiplied by a factor $(T/295)^{5/4}$ on reciprocal temperature $(1000/T)$ are depicted in Fig. 3. When $1000/T > 7$ (or $T < 40$ K), the relaxation time exponentially increases with temperature.

Temperature dependences of the variance of free electron density fluctuations and the Fermi energy for silicon with the donor impurity density 10^{15} cm^{-3} at $E_\mathrm{d} = 0.1$ eV are represented in Fig. 4(a). The maximum of the free carrier fluctuation variance is observed at the temperature $T_1 = 136$ K when $E_\mathrm{F}/E_\mathrm{d} = E_1/E_\mathrm{d} \approx 1.1$. However, $E_\mathrm{F} = E_\mathrm{d}$ at the temperature $T = T_\mathrm{F} = 116.5$ K, i.e. the maximum of the fluctuation variance appears when the Fermi level energy is below the donor level energy.

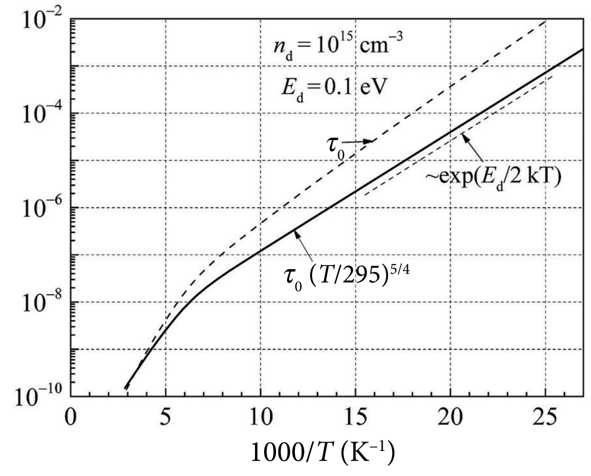

Fig. 3. Dependences of the free electron relaxation time τ_0 and the normalized relaxation time $\tau_0(T/295)^{5/4}$ on reciprocal temperature $(1000/T)$ for silicon with the donor density $n_\mathrm{d} = 10^{15}$ cm^{-3} at the energy level $E_\mathrm{d} = 0.1$ eV.

From Eq. (9) it follows that the relative variance of free electron number N fluctuations in the silicon sample with the number of donors N_d can be expressed as

$$\frac{\left\langle \Delta N^2 \right\rangle}{N_0} = \frac{N_\mathrm{d} - N_0}{2N_\mathrm{d} - N_0}. \tag{10}$$

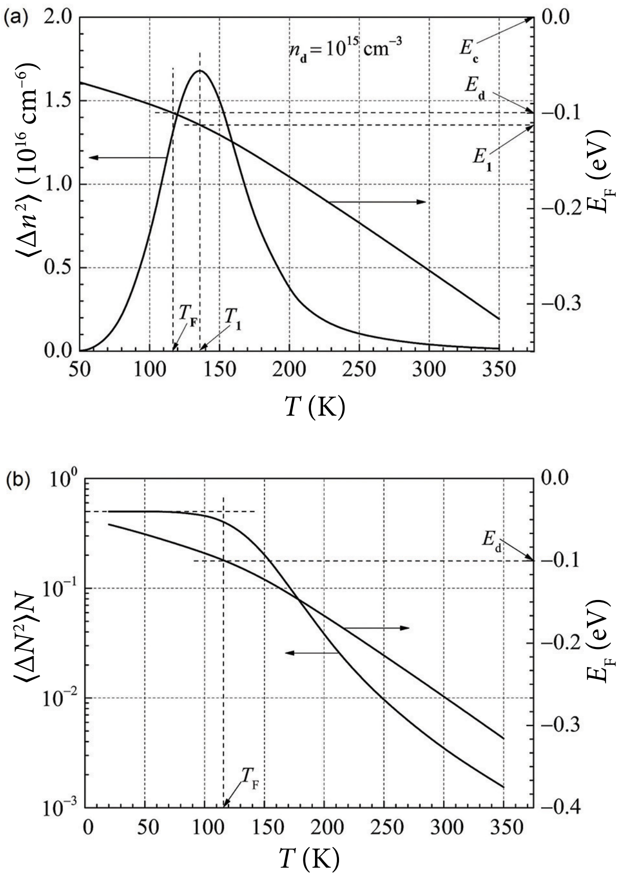

Fig. 4. Temperature dependences of (a) the variance of free electron density fluctuations $\langle \Delta n^2 \rangle$ and the Fermi energy E_F, and (b) the relative variance of free electron number fluctuations $\langle \Delta N^2 \rangle / N$ and the Fermi energy E_F for silicon with the donor density 10^{15} cm^{-3} at the energy level $E_\mathrm{d} = 0.1$ eV (there T_F is the temperature when the Fermi level energy coincides with the donor level energy, and T_1 is the temperature when the variance of free electron density fluctuations reaches the maximum value).

This relation $\langle \Delta N^2 \rangle / N_0 = 0.5$ when $N_0 \ll N_d$. The dependence of the relative variance of free electron number fluctuations on temperature is shown in Fig. 4(b), and it clearly illustrates that $\langle \Delta N^2 \rangle / N_0 = 0.5$ when the Fermi level energy is over the donor level energy. For random particles with Poisson's law of distribution, this ratio is equal to one. The smaller ratio for the electrons in the donor silicon is due to the degeneracy factor $\beta = 1/2$. For a silicon sample with one donor level $E_d = 0.1$ eV at temperature $T < 120$ K, the Fermi level energy is always over the donor level energy (Fig. 2). There the free electron density exponentially decreases with temperature decreasing (Fig. 2), while the relaxation time exponentially increases (Fig. 3). In this temperature range, the relative variance of free electron number fluctuations $\langle \Delta N^2 \rangle / N_0 = 0.5$. The maximum of the variance of free electron density fluctuations $\langle \Delta n^2 \rangle$ occurs when the Fermi level energy is about 10% below the donor level energy (Fig. 4(a)).

For the g-r noise spectrum calculation, it is convenient to present the results as a normalized power spectral density (multiplied by frequency):

$$\frac{S_N(f)}{N_0^2} \times f = 4 \frac{\langle \Delta n^2 \rangle}{n_0^2} \cdot \frac{f \tau_0}{1 + (\omega \tau_0)^2}. \tag{11}$$

The frequency dependences of the normalized power spectral density of free electron number fluctuations for a silicon sample at different temperatures are presented in Fig. 5.

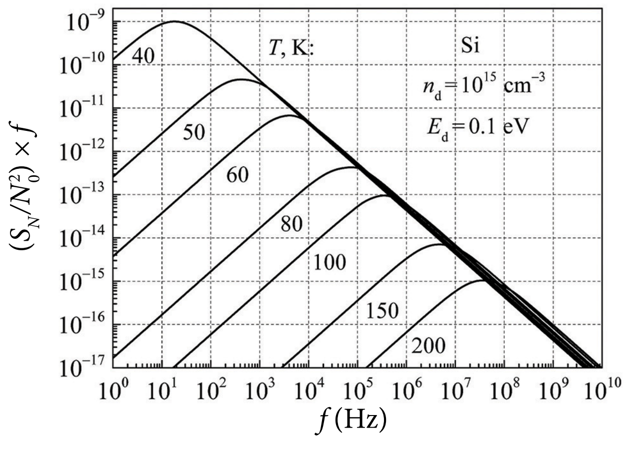

Fig. 5. Normalized spectra of free electron number fluctuations for a silicon sample with the donor density $n_d = 10^{15}$ cm^{-3} at the energy level $E_d = 0.1$ eV at different temperatures.

The maximum of the normalized spectral density of free electron number fluctuations decreases and shifts to higher frequencies with increasing temperature (Fig. 5). The peak values are observed at frequencies $f_0 = 1/(2\pi\tau_0)$. By presenting the measurement results in this way, it is very easy to determine the relaxation time τ_0.

2.2. G-r properties in silicon with various shallow donor densities

At high shallow donor densities, donor ions interact with each other, causing their wave functions to overlap and the donor energy level to split [21]. This results in a decreased ionization energy and the formation of the band tail at the bottom of the conduction band rather than a well-defined band edge. Taking into account that the donor atoms replace the silicon atoms in the lattice [22], the donor levels cannot be considered as additional levels to the levels of the conduction band. These donor levels partly overlap with the energy levels in the conduction band, and when $E_d = 0$, the donor energy states completely overlap the conduction band, and all valence electrons are in the conduction band. The decrease of the shallow donor level energy ($E_{d0} = 0.05$ eV) with the increase in donor density for the silicon sample is presented in Fig. 6(a) and can be described as [20]

$$E_d = 0.05 - 2.33 \cdot 10^{-8}\, n_d^{1/3}, \text{ eV}. \tag{12}$$

Temperature dependences of the free electron density and the Fermi level energy at three different shallow donor densities, 10^{13}, 10^{15} and 10^{17} cm^{-3}, in silicon are presented in Fig. 6(b). The dots A_1, A_2 and A_3 represent the free electron densities at temperatures T_{F1}, T_{F2} and T_{F3}, respectively, when the Fermi level energy coincides with the donor level energy and Eq. (12) is taken into account. Similarly, the dots B_1, B_2 and B_3 denote the Fermi level energies when they coincide with the donor level energies and Eq. (12) is also taken into account. Therefore, it is clear that the relation $n_0 = (1/3)\, n_d$ is perfectly fulfilled for various presented donor densities when the Fermi level energy coincides with the donor level energy.

Temperature dependences of the relative free electron density n_0/n_d for three different donor densities are shown in Fig. 7(a). Usually, it is

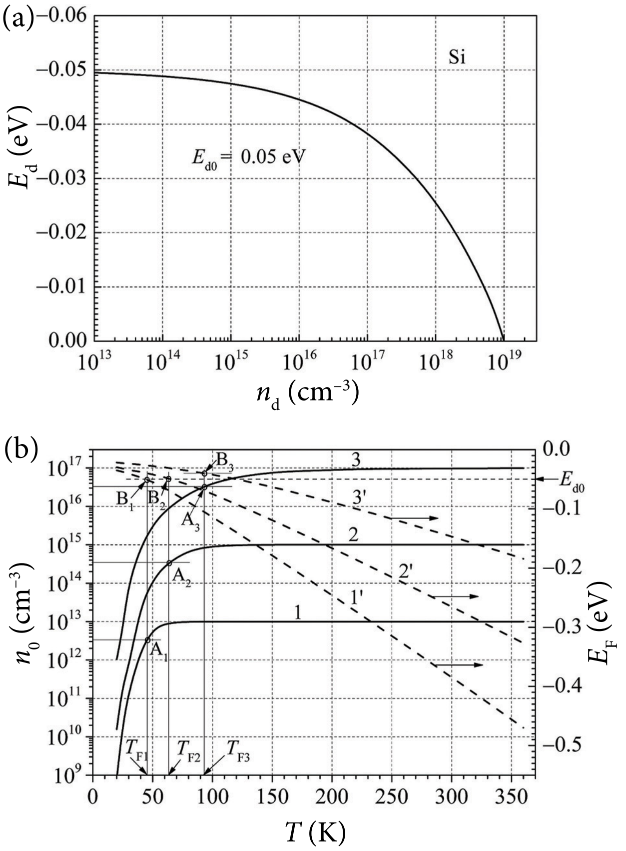

Fig. 6. (a) The decrease of the shallow donor level energy E_d with the increase in donor density at room temperature; (b) temperature dependences of the free electron density (left scale) and the Fermi level energy (right scale) at three different donor densities, in cm^{-3}: 1 at 10^{13}; 2 at 10^{15}; 3 at 10^{17} (the donor energy decrease is taken into account).

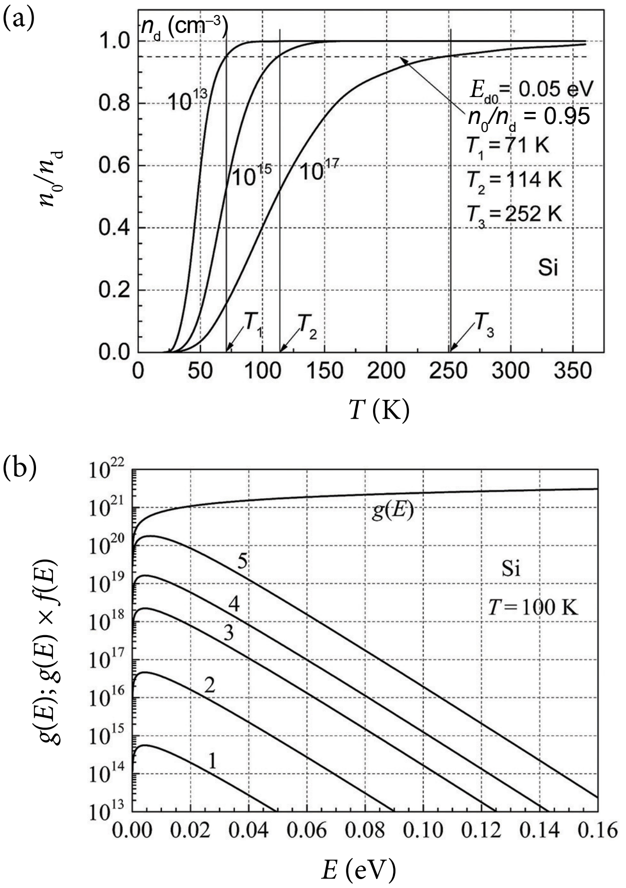

Fig. 7. (a) Temperature dependences of the relative density of the free electrons n_0/n_d for three different donor densities; (b) dependences of the density of states $g(E)$ and the states filling function $g(E)f(E)$ on energy at $T = 100$ K in silicon for various donor densities, cm^{-3}: 1 for 10^{13} (−0.114 eV); 2 for 10^{15} (−0.075 eV); 3 for 10^{17} (−0.0423 eV); 4 for 10^{18} (−0.0252 eV); 5 for 10^{19} (−0.00131 eV), here in brackets the donor energy values taking into account the donor level energy decrease by Eq. (12) are presented.

considered that all donors with a shallow energy level of 0.05 eV are completely ionized at temperatures above liquid nitrogen. As Fig. 7(a) indicates, the requirement for 95% of donors to be ionized at temperatures above liquid nitrogen is only fulfilled at low (≤10^{13} cm^{-3}) donor densities. This ionization level for donor density of 10^{15} cm^{-3} is only achieved at temperatures above 114 K, and for 10^{17} cm^{-3} only at temperatures above 252 K. Thus, the statement that all shallow donors are ionized at temperatures above liquid nitrogen should be used with extreme caution. What determines that not all donors are ionized at higher shallow donor density? The higher the shallow donor density, the lower the relative free electron density (Fig. 7(a)): only 20% of donors are ionized at the temperature of liquid nitrogen when donor density is 10^{17} cm^{-3}. The dependence of the conduction band states filling function $g(E)f(E)$ on energy at $T = 100$ K is presented in Fig. 7(b). At low donor densities, electrons fill energy states near the bottom of the conduction band. However, for higher donor densities, some electrons must be excited to the higher energy levels of the conduction band because the energy levels near the bottom of the conduction band are occupied.

Temperature dependence of the free electron density for high shallow donor densities in silicon, which is estimated by Eq. (1), is depicted in Fig. 8(a). The Hall effect measurement data show that all donors are ionized when donor density is 10^{19} cm^{-3} [20]. Thus, Eq. (1) incorrectly describes the free electron density at high donor densities in

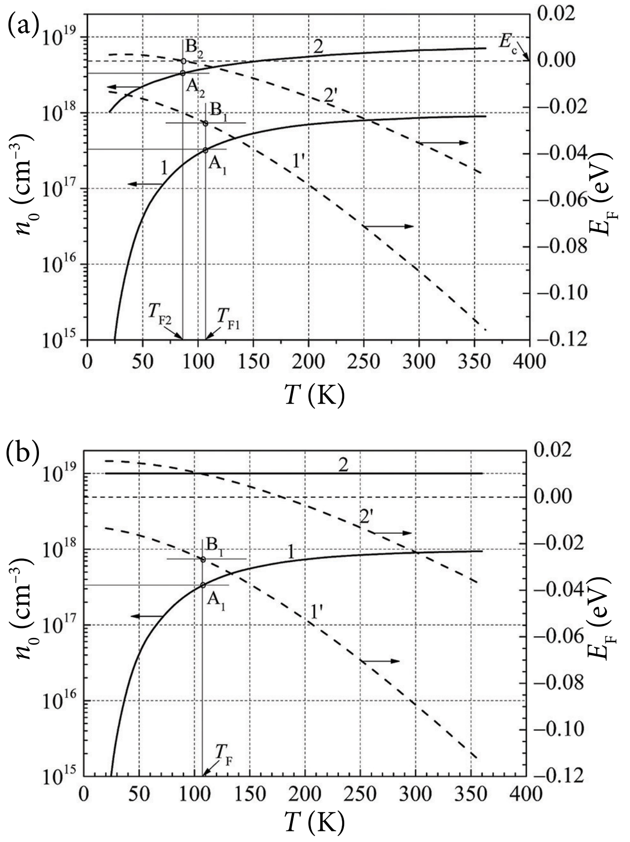

Fig. 8. (a) Temperature dependences of the free electron density (left scale) and the Fermi level energy (right scale) for high densities of shallow donors, in cm^{-3}: 1 for 10^{18} (E_d = 0.025 eV); 2 for 10^{19} (E_d = 0) calculated by Eq. (1); (b) the same as in Fig. 8(a), but calculated by Eq. (13). Dots A$_1$ and A$_2$ on curves $n_0(T)$ show the free electron densities when the Fermi level energy coincides with the donor level energy (denoted by dots B$_1$ and B$_2$).

silicon. Then, the expression for the free electron density can be estimated as

$$n_0 = n_d - n_d^0 = n_d - \frac{n_d[1-\exp(-\varepsilon_d)]}{1+\beta\exp(-\varepsilon_d-\eta)} = \frac{n_d(1+\beta^{-1}\exp\eta)}{1+\beta^{-1}\exp(\varepsilon_d+\eta)} = N_c F_{1/2}(\eta), \quad (13)$$

where the probability multiplier $[1-\exp(-\varepsilon_d)]$ shows that at $E_d = 0$, i.e. the donor energy states overlap with the conduction band levels. Considering that every donor changes the silicon atom (substitutional impurity) [22] and that donor levels are formed as decoupled levels from the conduction band, the total density of states due to doping changes weakly [20]. It is confirmed by electronic heat measurements of silicon with a high donor doping [23].

The temperature dependence of the free electron density for high donor densities estimated by Eq. (13) is presented in Fig. 8(b) and it is in good agreement with the experimental data [24]. Equation (13) is valid for any donor density with a low or high donor level energy. Equations (1) and (13) completely coincide at shallow donor densities ≤10^{17} cm^{-3}. To evaluate the density of the free electrons for any donor density, it is convenient to have an analogous expression for the function $F_{1/2}(\eta)$,

$$F_{1/2} = \exp(\eta)/\alpha, \quad (14)$$

where the parameter

$$\alpha = 1 + 0.28\exp(0.65\eta) + 0.09\exp(0.9\eta). \quad (15)$$

Dependences of the functions $\exp(\eta)$, $F_{1/2}(\eta)$ and α on η are presented in Fig. 9(a). The difference

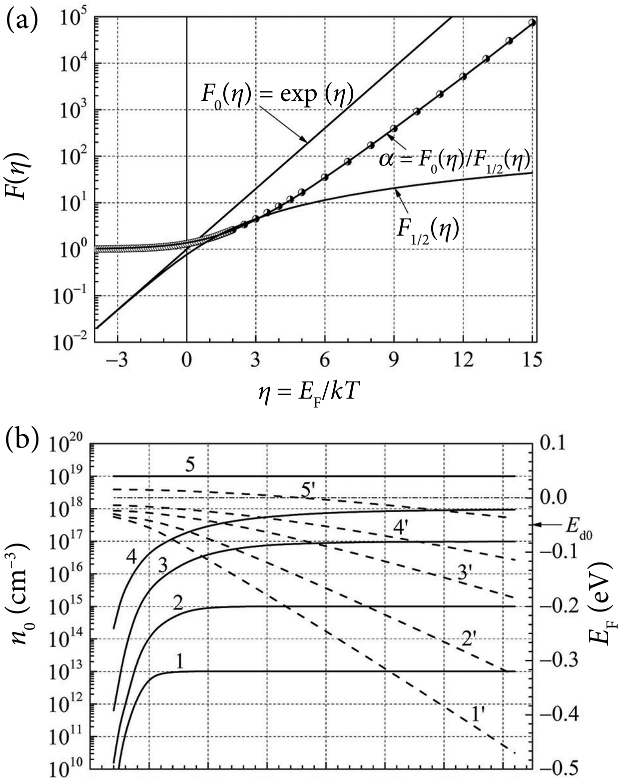

Fig. 9. (a) Dependences of functions $\exp\eta$, $F_{1/2}(\eta)$ and α on η (dots on α curve denote the calculated ratio $\exp\eta/F_{1/2}(\eta)$, solid α line is the approximation by Eq. (15)); (b) temperature dependences of the free electron density (calculated by Eq. (13), left scale) and the Fermi level energy (right scale) for various shallow donor densities n_d, in cm^{-3}: 1 for 10^{13}; 2 for 10^{15}; 3 for 10^{17}; 4 for 10^{18}; 5 for 10^{19} (here Eq. (12) has also been taken into account).

between the approximation curve (Eq. (15)) and the ratio $\exp(\eta)/F_{1/2}(\eta)$ does not exceed 5% in the whole η range ($\eta \leq 15$). At η values below –3, the ratio $\exp(\eta)/F_{1/2}(\eta) = 1$. Temperature dependences of the free electron density calculated by Eq. (13) for five different donor densities are presented in Fig. 9(b). The obtained data are in good agreement with the experimental Hall effect measurement results [20]. The Fermi level energy for all donor densities has negative values, but for $n_d = 10^{19}$ cm^{-3} it becomes positive at temperatures below 150 K.

Temperature dependences of the variance of electron density fluctuations and the relative variance of free electron number fluctuations $\langle \Delta N^2 \rangle / N_0$ at three different densities of shallow donors are depicted in Fig. 10. The temperature dependence of the variance of free electron density fluctuations is characterized by a maximum, which strongly increases (about four orders of magnitude) with increasing donor density and shifts in the temperature range from 50 to 125 K (Fig. 10(a)). The relative variance of free electron number fluctuations $\langle \Delta N^2 \rangle / N_0$ at low temperatures is equal to 0.5, and this value does not depend on the donor density but decreases strongly at higher temperatures (Fig. 10(b)). It can be pointed out that the value 0.5 reflects a situation where the free electron fluctuations obey Poisson's law of distribution.

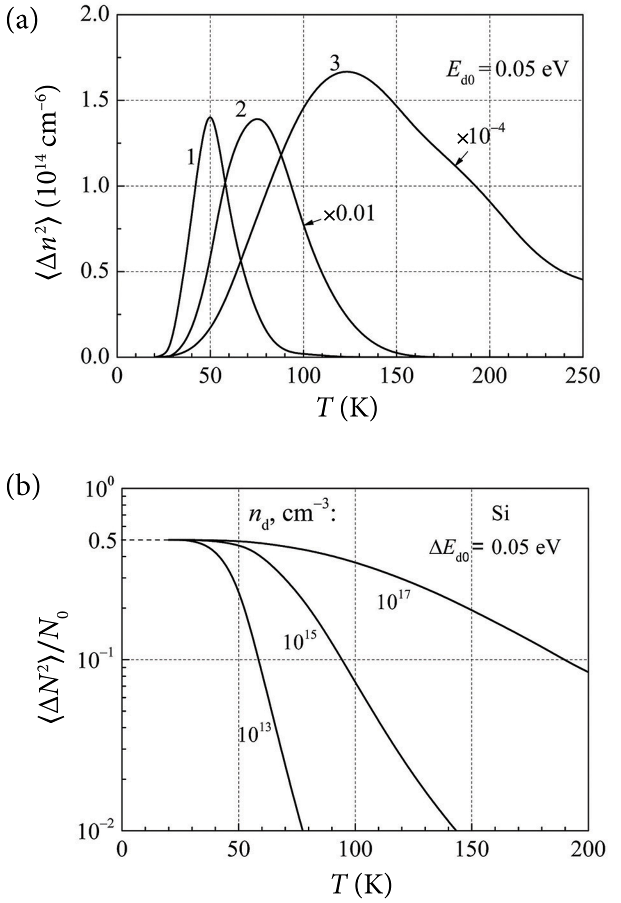

Fig. 10. Temperature dependences of (a) the variance of the free electron density $\langle \Delta n^2 \rangle$ and (b) the relative variance of free electron number fluctuations $\langle \Delta N^2 \rangle / N_0$ at three different densities of shallow donors ($E_{d0} = 0.05$ eV), in cm^{-3}: 1 at 10^{13}; 2 at 10^{15}; 3 at 10^{17} (here Eq. (12) has also been taken into account).

2.3. Temperature properties of the generation-recombination noise in silicon with two donor levels in the presence of acceptors

A donor doped silicon sample of volume $V = 0.01$ cm^3 with the shallow donor density $n_{1d} = 2{\cdot}10^{13}$ cm^{-3} at the energy level $E_{1d} = 0.05$ eV and with the deep donor density $n_{2d} = 3{\cdot}10^{14}$ cm^{-3} at the level $E_{2d} = 0.25$ eV is partially compensated by acceptors with the density $n_a = 1.5{\cdot}10^{13}$ cm^{-3} (Fig. 11).

In the general case, the electron generation and recombination from the shallow donor level E_{1d} in the silicon partially compensated by acceptors can be described as

$$g_1(n) = a_1 n^0_{1d} = a_1(n_{1d} - n_a - n^+_{1d}), \qquad (16)$$

$$r_1(n) = \gamma n(n^+_{1d} + n_a) = \gamma(n^+_{1d} + n^+_{2d})(n^+_{1d} + n_a), \qquad (17)$$

where n^0_{1d} is additionally decreased due to the capture of electrons by acceptors; the parameter a_1

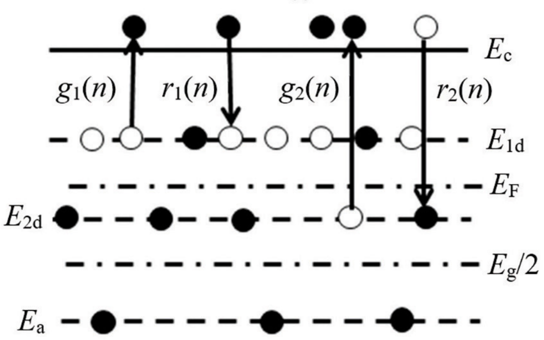

Fig. 11. Schematic illustration of the generation-recombination process in silicon with two donor levels and with one acceptor level.

can be found from the equilibrium condition $g_1(n_0) = r_1(n_0)$:

$$a_1 = \frac{\gamma n_0 (n_{1\mathrm{d}}^+ + n_\mathrm{a})}{n_{1\mathrm{d}} - n_{1\mathrm{d}}^+ - n_\mathrm{a}}. \tag{18}$$

Then, the electron relaxation time of the g-r process between the shallow donor level and conduction band can be estimated as

$$\tau_1 = \frac{1}{r'_1(n) - g'_1(n)} = \frac{(n_{1\mathrm{d}} - n_\mathrm{a} - n_{1\mathrm{d}}^+)/\gamma}{n_{1\mathrm{d}}(n_0 + n_{1\mathrm{d}}^+) - n_{1\mathrm{d}}^+(n_{1\mathrm{d}}^+ + n_\mathrm{a})}, \tag{19}$$

and the free electron variance produced by the shallow donors is

$$\langle \Delta n_1^2 \rangle = g_1(n_0)\tau_1/V = \gamma n_0 (n_{1\mathrm{d}}^+ + n_\mathrm{a})\tau_1/V. \tag{20}$$

The electron generation and recombination from the deep donor level $E_{2\mathrm{d}}$ can be described in a similar way:

$$g_2(n) = a_2 n_{2\mathrm{d}}^0 = a_2(n_{2\mathrm{d}} - n_{2\mathrm{d}}^+), \tag{21}$$

$$r_2(n) = \gamma n n_{2\mathrm{d}}^+ = \gamma(n_{1\mathrm{d}}^+ + n_{2\mathrm{d}}^+ - n_\mathrm{a})n_{2\mathrm{d}}^+, \tag{22}$$

$$a_2 = \frac{\gamma n_0 n_{2\mathrm{d}}^+}{n_{2\mathrm{d}} - n_{2\mathrm{d}}^+}, \tag{23}$$

$$\tau_2 = \frac{1}{r'_2(n) - g'_2(n)} = \frac{(n_{2\mathrm{d}} - n_{2\mathrm{d}}^+)/\gamma}{(n_0 + n_{2\mathrm{d}}^+)(n_{2\mathrm{d}} - n_{2\mathrm{d}}^+) + n_0 n_{2\mathrm{d}}^+}, \tag{24}$$

$$\langle \Delta n_2^2 \rangle = g_2(n_0)\tau_2/V = \gamma n_0 n_{2\mathrm{d}}^+ \tau_2/V. \tag{25}$$

Considering that the effective density of donors $n_{1\mathrm{d}}$ decreases by the value n_a due to the number of electrons captured by acceptors, the average free electron density can be calculated by the following expression:

$$n_0(T) = \frac{n_{1\mathrm{d}} - n_\mathrm{a}}{1 + \beta^{-1}\exp(\varepsilon_{1\mathrm{d}} + \eta)} + \frac{n_{2\mathrm{d}}}{1 + \beta^{-1}\exp(\varepsilon_{2\mathrm{d}} + \eta)} = N_\mathrm{c} F_{1/2}(\eta). \tag{26}$$

Temperature dependences of the average density of free electrons are shown in Fig. 12(a). Dependences of the relaxation times from the donor levels $E_{1\mathrm{d}}$ and $E_{2\mathrm{d}}$ on the reciprocal temperature (1000/T) are presented in Fig. 12(b). The relaxation time from the shallow donor level exponentially increases when temperature $T < 50$ K (or $1000/T > 20$). The excitation of free electrons from the deep donor level $E_{2\mathrm{d}}$ is noticeable only at $T > 100$ K (or at $1000/T < 10$). Due to the compensation by acceptors, the free electron density decreases by the amount of the acceptor density (plateau region in Fig. 12(a)). However, as Fig. 12(b) indicates, the relaxation time of free electrons experiences small changes due to the compensation by acceptors.

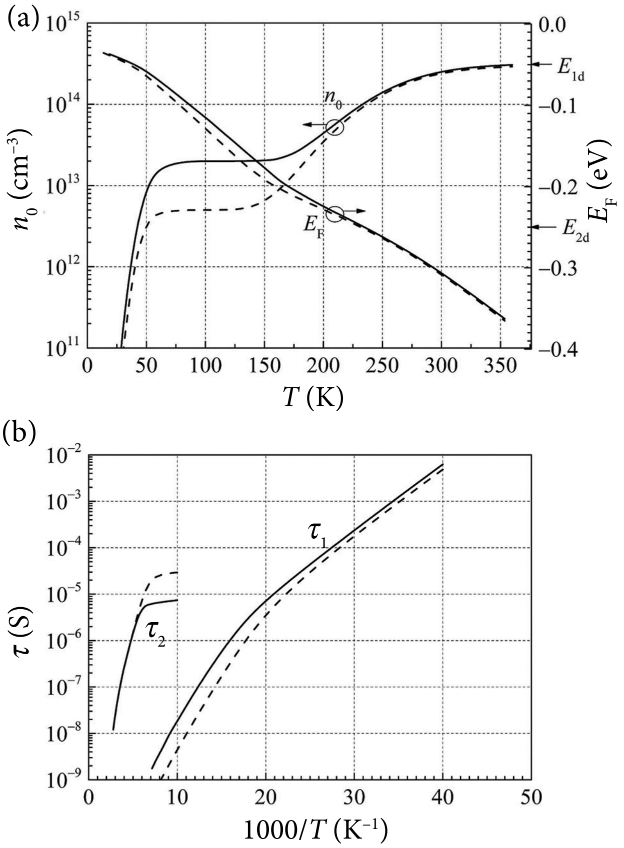

Fig. 12. (a) Temperature dependences of the free electron density (left scale) and the Fermi energy (right scale), and (b) dependences of the relaxation times of free electrons from the donor levels on reciprocal temperature (1000/T) in the partially compensated by acceptors $n_\mathrm{a} = 1.5\cdot10^{13}$ cm^{-3} (dashed lines) and without the compensation by acceptors (solid lines) silicon sample with the shallow donor density $n_{1\mathrm{d}} = 2\cdot10^{13}$ cm^{-3} at the energy level $E_{1\mathrm{d}} = 0.05$ eV, and with the deep donor density $n_{2\mathrm{d}} = 3\cdot10^{14}$ cm^{-3} at the energy level $E_{2\mathrm{d}} = 0.25$ eV.

Temperature dependences of the total variance of free electron density fluctuations $\langle \Delta n^2 \rangle$ (left scale) and the Fermi level energy (right scale) are represented in Fig. 13(a) (solid line is for the silicon without the compensation by acceptors; dashed line is for the silicon in the presence of acceptors with density $n_a = 1.5\cdot10^{13}$ cm^{-3}). The solid line branch of $\langle\Delta n^2\rangle$ below $T = 75$ K is caused by the shallow donor level E_{1d}, and when $T > 75$ K, the branch of $\langle\Delta n^2\rangle$ temperature dependence is due to the deep donor level E_{2d}. The minimum in the temperature dependence of $\langle\Delta n^2\rangle$ occurs at $T \approx 120$ K. The compensation by acceptors significantly shifts the minimum to $T \approx 50$ K, and the magnitude of this value decreases by about five orders (Fig. 13(a)). Temperature dependences of the relative variance of the free electron number fluctuations $\langle\Delta N^2\rangle/N_0$ are shown in Fig. 13(b) (the meaning of solid and dashed lines is the same as in Fig. 13(a)). The solid line branch of $\langle\Delta N^2\rangle/N_0$ at $T < 120$ K is caused by the fluctuations of the free electron number from the shallow donor level, and the branch of $\langle\Delta N^2\rangle/N_0$ at $T > 120$ K is caused by the fluctuations of the free electron number from the deep donor level. The temperature dependence of $\langle\Delta N^2\rangle/N_0$ has a minimum at $T \approx 125$ K for the silicon without acceptors. The compensation by acceptors also noticeably shifts this minimum to the lower temperature $T \approx 65$ K, and the magnitude of the minimum value decreases by about five orders. When the Fermi level energy exceeds E_{1d}, then $\langle\Delta N^2\rangle/N_0 \approx 0.5$ for the non-compensated silicon as in the case of silicon with one donor level.

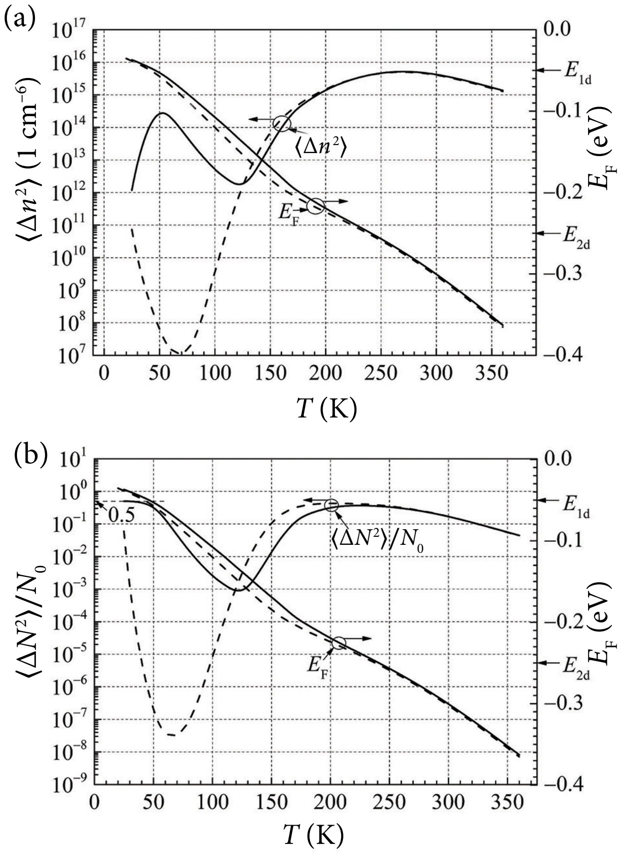

Fig. 13. Temperature dependences of (a) the total variance of free electron density fluctuations $\langle\Delta n^2\rangle$, and (b) the relative variance of free electron number fluctuations $\langle\Delta N^2\rangle/N_0$ in the partially compensated by acceptors $n_a = 1.5\cdot10^{13}$ cm^{-3} (dashed lines) and in the acceptors free (solid lines) silicon sample with the shallow donor density $n_{1d} = 2\cdot10^{13}$ cm^{-3} at the energy level $E_{1d} = 0.05$ eV, and with the deep donor density $n_{2d} = 3\cdot10^{14}$ cm^{-3} at the energy level $E_{2d} = 0.25$ eV.

The normalized spectral density of free electron number fluctuations in this case can be described as

$$\frac{S_N(f)}{N_0^2}\times f = 4\frac{\langle\Delta n_1^2\rangle}{n_0^2}\cdot\frac{f\tau_1}{1+(\omega\tau_1)^2} + 4\frac{\langle\Delta n_2^2\rangle}{n_0^2}\cdot\frac{f\tau_2}{1+(\omega\tau_2)^2}. \qquad (27)$$

The calculation results by Eq. (27) for the silicon without the compensation by acceptors are presented in Figs. 14–15. The noise maximum in Fig. 14(a) abruptly decreases with temperature and shifts to the higher frequencies due to the electron number fluctuations in the shallow donor level E_{1d}. The same tendency has been observed in the silicon sample with one donor level (Fig. 5). However, changes in the temperature dependence of the normalized spectra of free electron number fluctuations caused by the donor level E_{2d} (Fig. 14(b)) are different: in the temperature range from 120 to 200 K, the noise maximum increases and slowly shifts to the higher frequencies with temperature, and then, at higher temperatures, the maximum decreases in the same way as in Fig. 14(a). The g-r noise from the shallow donor level prevails at temperatures $T < 100$ K, while at $T > 140$ K, the total fluctuations are caused by the donor level with energy $E_{2d} = 0.25$ eV.

The comparison of the data presented in Fig. 14(a, b) reveals that in the latter case the position of the maximum of the normalized resistance fluctuation spectrum very weakly depends on the temperature in the range from 140 to

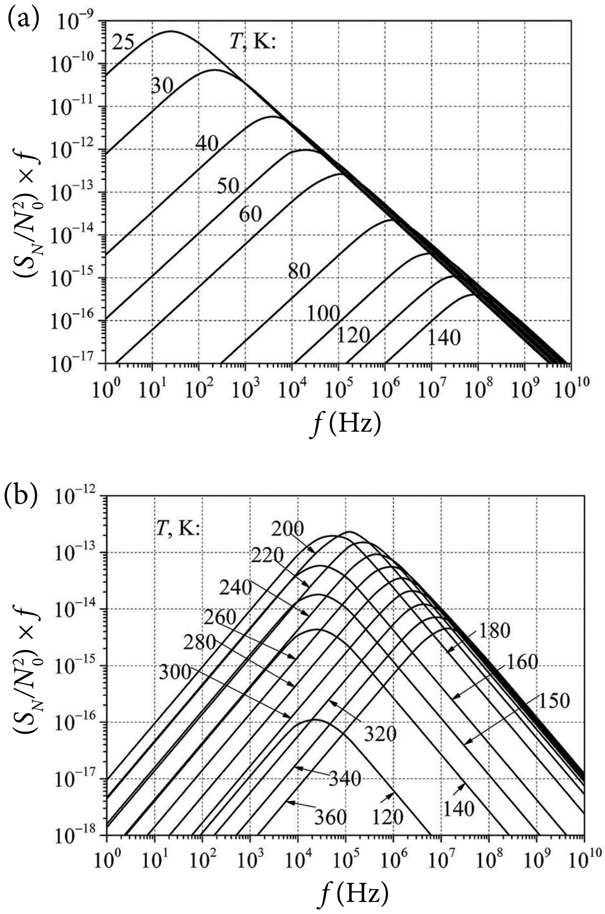

Fig. 14. The normalized spectra of free electron number fluctuations caused by (a) the shallow donor level E_{1d} = 0.05 eV with donor density $n_{1d} = 2\cdot10^{13}$ cm^{-3} and (b) the donor level E_{2d} = 0.25 eV with the donor density $n_{2d} = 3\cdot10^{14}$ cm^{-3} at various temperatures for the silicon sample without the compensation by acceptors.

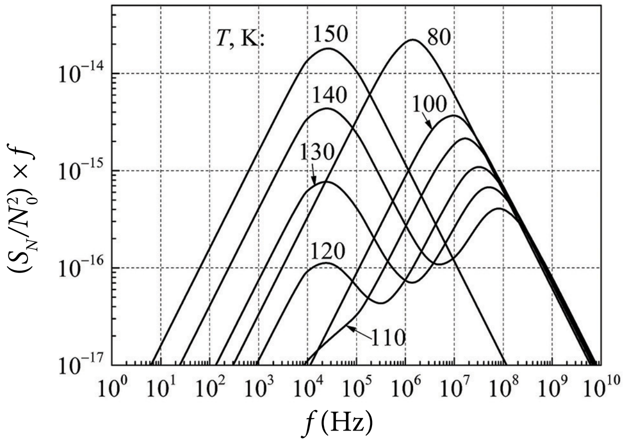

Fig. 15. The normalized spectra of free electron number fluctuations for the acceptor free silicon sample with two donor levels (E_{1d} = 0.05 and E_{2d} = 0.25 eV) in the intermediate temperature range (100–150) K.

200 K. The normalized spectra of free electron number fluctuations due to both donor levels when the contribution to noise spectra of both donor levels is of the same order in the intermediate temperature range (100–150) K are presented in Fig. 15. The first peak of the noise spectrum is observed in the frequency interval from 10 to 100 kHz and weakly depends on the temperature in the intermediate temperature range. However, the position of the second noise peak changes from 1 to 100 MHz in the temperature interval from 80 to 140 K.

Consider the noise spectra of the same sample including the compensation by acceptors. The spectral density of the g-r noise in this case can be described as

$$\frac{S_N(f)}{N_0^2} = 4\frac{\left\langle \Delta n_1^2\right\rangle}{n_0^2}\cdot\frac{\tau_1}{1+(\omega\tau_1)^2} + 4\frac{\left\langle \Delta n_2^2\right\rangle}{n_0^2}\cdot\frac{\tau_2}{1+(\omega\tau_2)^2}. \tag{28}$$

The calculation results obtained by Eq. (28) are presented in Fig. 16. The dashed curves are due to the free electron number fluctuations from the shallow donor level E_{1d}. The dashed arrows show the changes of the g-r noise spectra when temperature increases. The solid curves represent the noise spectra caused by the free electron number fluctuations from the deep donor level E_{2d}.

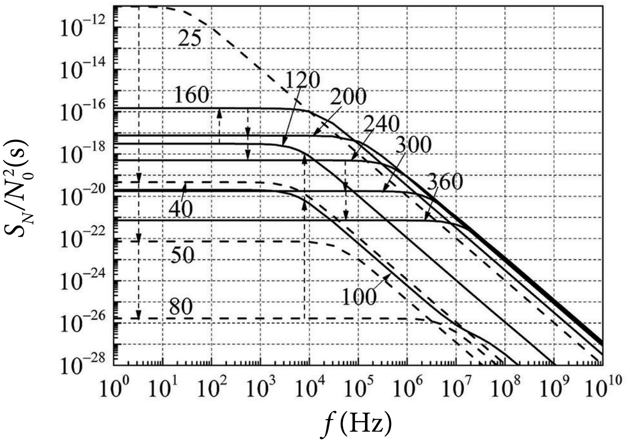

Fig. 16. The relative g-r noise spectra S_N/N_0^2 for the partially compensated by acceptors ($n_a = 1.5\cdot10^{13}$ cm^{-3}) silicon sample with the shallow donor density $n_{1d} = 2\cdot10^{13}$ cm^{-3} at the energy level E_{1d} = 0.05 eV and with the deep donor density $n_{2d} = 3\cdot10^{14}$ cm^{-3} at the energy level E_{2d} = 0.25 eV at various temperatures.

The g-r noise spectra due to the deep level donors always exceed the g-r noise level caused by the shallow donors at temperatures $T > 100$ K.

The results of the normalized g-r noise spectra calculated by Eq. (27) are presented in Fig. 17.

The comparison of the calculation results for silicon samples with the same donor densities, but in one case without the compensation by acceptors (Fig. 14) and in another with partial compensation by acceptors, shows that the noise spectra presented in Fig. 17(a) and caused by the shallow donors have noticeable changes due to the influence of acceptors and free electrons from the deep donor level at temperatures $T > 80$ K. The noise spectra caused by the deep donors (Fig. 17(b)) have almost the same pattern as in Fig. 14(b).

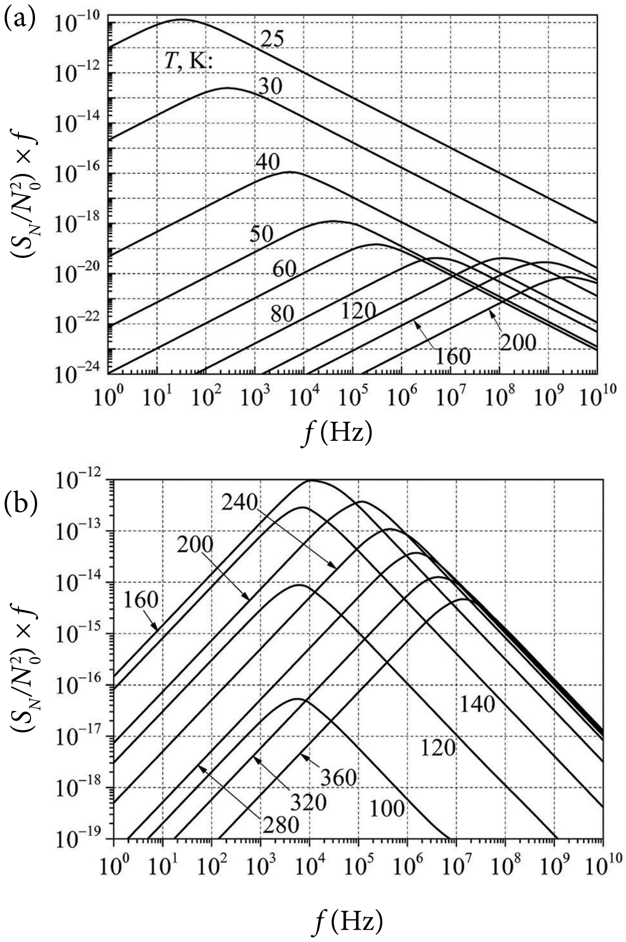

Fig. 17. The normalized g-r noise spectra caused by (a) the shallow donor level $E_{1d} = 0.05$ eV with the donor density $n_{1d} = 2\cdot10^{13}$ cm^{-3}, and (b) the deep donor level $E_{2d} = 0.25$ eV with the donor density $n_{2d} = 3\cdot10^{14}$ cm^{-3} in the partially compensated by acceptors ($n_a = 1.5\cdot10^{13}$ cm^{-3}) silicon sample.

3. Conclusions

The generation-recombination processes in silicon have been analyzed in the temperature range from 20 to 360 K. It is demonstrated that the electron density strongly depends on temperature: the shallow donors with a density smaller than 10^{13} cm^{-3} are completely ionized at liquid nitrogen and higher temperatures, while only 20% of donors are ionized when the density of shallow donors is about 10^{17} cm^{-3} at liquid nitrogen temperature. The electrons fill the energy states near the bottom of the conduction band when donor density is low, but for the higher donor densities, some electrons must be excited to the higher energy levels of the conduction band, because the energy levels near the bottom of the conduction band are occupied, and the higher temperature is needed for excitation. It is also shown that in the transient region to the degenerate state, the standard Eq. (1) does not fully accurately describe the free electron density dependence on the shallow donor density. The revised formula (Eq. (13)), which is valid at any shallow donor density, is presented.

The maximum of the variance of free electron density fluctuations for a silicon sample with shallow donors strongly increases (about four orders of magnitude) with increasing donor density and shifts in the temperature range from 50 to 125 K. The relative variance of free electron number fluctuations ($\langle \Delta N^2 \rangle / N_0$) is always equal to 0.5 at low temperatures and reflects the situation when the free electron fluctuations obey the Poisson's law.

The normalized spectra of the generation-recombination noise in silicon with shallow and deep donor levels are presented in the frequency interval from 1 to 10^{10} Hz and the temperature range from 25 to 360 K. The comparison between the g-r noise spectra in silicon with and without the partial compensation by acceptors is also presented.

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GENERACINIS-REKOMBINACINIS TRIUKŠMAS IR KITOS LEGIRUOTO SILICIO YPATYBĖS PLAČIAME TEMPERATŪROS INTERVALE

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Santrauka

Silicio generacinio-rekombinacinio triukšmo charakteristikos yra ištirtos temperatūros intervale nuo 25 iki 360 K. Ypač didelis dėmesys skirtas siliciui su sekliomis donorinėmis priemaišomis, esant labai plačiam donorų tankiui. Parodyta, kad laisvųjų elektronų tankis stipriai priklauso nuo temperatūros net sekliųjų donorų (donorinio lygmens energija lygi 0,05 eV) atveju. Seklieji donorai, esant jų tankiui mažesniam nei 10^{13} cm^{-3}, skysto azoto temperatūroje yra visiškai jonizuoti, o sekliųjų donorų, kurių tankis yra apie 10^{17} cm^{-3}, tik apie 20 % yra jonizuoti. Esant mažam sekliųjų donorų tankiui, laisvieji elektronai užpildo energijos lygmenis, esančius arti laidumo juostos dugno. Didėjant donorų tankiui, dalis elektronų turi būti sužadinami į aukštesnius laidumo juostos energijos lygmenis, todėl jiems sužadinti reikalinga aukštesnė temperatūra. Taip pat parodyta, kad pereinamojoje link išsigimusių elektroninių dujų srityje standartinis sąryšis (1) ne visai tiksliai parodo laidumo juostos elektronų tankio priklausomybę nuo sekliųjų donorų tankio. Pateikta patikslinta išraiška (13), kuri galioja esant bet kokiam sekliųjų donorų tankiui.

Minėtos sekliųjų donorų ypatybės pasireiškia ir generacinio-rekombinacinio triukšmo charakteristikose. Generacinio-rekombinacinio triukšmo dispersijos maksimumas dėl laisvųjų elektronų tankio fliuktuacijų silicyje, kylant sekliųjų donorų tankiui nuo 10^{13} iki 10^{17} cm^{-3}, stipriai didėja (apie 4 eiles), ir šis maksimumas pasislenka temperatūros skalėje nuo 50 iki 125 K. Parodyta, kad laisvųjų elektronų skaičiaus N fliktuacijų dispersija ($\langle \Delta N^2 \rangle / N_0$) žemoje temperatūroje visada lygi 0,5, ir tai rodo, kad žemoje temperatūroje laisvųjų elektronų skaičiaus fliuktuacijos apibūdinamos Puasono (Poisson) tikimybių pasiskirstymo dėsniu.

Normuoto generacinio-rekombinacinio triukšmo spektrų $[S_N(f)/N_0^2 \times f]$ dažninės priklausomybės yra pateiktos intervale nuo 1 iki 10^{10} Hz temperatūros srityje nuo 25 iki 360 K. Taip pat pateiktos silicio generacinio-rekombinacinio triukšmo charakteristikos, esant dviem donoriniams lygmenims, įskaitant kompensavimą akceptoriais.